ÇUKUROVA UNIVERSITY
INSTITUTE OF NATURAL AND APPLIED SCIENCES

MSc THESIS

Çağrı UZAY

A COMPARISON OF APPROACHES TO INVOLUTE SPUR GEAR DESIGN

DEPARTMENT OF MECHANICAL ENGINEERING

ADANA, 2014

ÇUKUROVA UNIVERSITY
INSTITUTE OF NATURAL AND APPLIED SCIENCES

A COMPARISON OF APPROACHES TO INVOLUTE SPUR GEAR DESIGN

Çağrı UZAY

MSc THESIS

DEPARTMENT OF MECHANICAL ENGINEERING

We certify that the thesis titled above was reviewed and approved for the award of degree of the Master of Science by the board of jury on 25/08/2014.

……….……................	…………………………………..	....
Prof. Dr. Necdet GEREN	Prof. Dr. Melih BAYRAMOĞLU	Assoc. Prof. Dr. Hakan YAVUZ
SUPERVISOR	MEMBER	MEMBER

This MSc Thesis is written at the Department of Mechanical Engineering of Institute of Natural And Applied Sciences of Çukurova University.
Registration Number:

Prof. Dr. Mustafa GÖK
Director
Institute of Natural and Applied Sciences

ABSTRACT

MSc THESIS

A COMPARISON OF APPROACHES TO INVOLUTE SPUR GEAR DESIGN

Çağrı UZAY

ÇUKUROVA UNIVERSITY
INSTITUTE OF NATURAL AND APPLIED SCIENCES
DEPARTMENT OF MECHANICAL ENGINEERING

Supervisor : Prof. Dr. Necdet GEREN
Year: 2014, Pages: 179
Jury : Prof. Dr. Necdet GEREN
: Prof. Dr. Melih BAYRAMOĞLU
: Assoc. Prof. Dr. Hakan YAVUZ

This thesis meets a need of selecting and using appropriate involute spur gear design approaches for all designers including the expert designers and novice learners who are practicing a spur gear design. Five design approaches with different level of difficulty, including the most commonly used machine elements textbooks, national and international standards were selected for comparison of design results. The results of each approach were analyzed by using a finite element method, ANSYS. And the loss or gain obtained from each of the approach was determined and results were given comparatively considering the gear failures criteria, speed ratios and power transmission ranges. Useful outputs, practical curves and charts were introduced to select the appropriate design approach. In addition to this, the study provides conversion factors which may be used to multiply the results of simple gear design approaches to ANSI/AGMA standards or in any of the five selected one. It also offers the best approach for students and designers who aim to optimize the gear design.

Key Words: Spur gear, Design approaches, Design outputs, Comparison

ÖZ

YÜKSEK LİSANS TEZİ

EVOLVENT DÜZ DİŞLİ TASARIM YAKLAŞIMLARININ KARŞILAŞTIRILMASI

Çağrı UZAY

ÇUKUROVA ÜNİVERSİTESİ
FEN BİLİMLERİ ENSTİTÜSÜ
MAKİNE MÜHENDİSLİĞİ ANABİLİM DALI

Danışman : Prof. Dr. Necdet GEREN
Yıl: 2014, Sayfa: 179
Jüri : Prof. Dr. Necdet GEREN
: Prof. Dr. Melih BAYRAMOĞLU
: Doç. Dr. Hakan YAVUZ

Bu tez, bir düz dişli tasarımı ile uğraşan uzman tasarımcılar ve acemi öğrenciler dahil tüm tasarımcılar için uygun evolvent düz dişli tasarım yaklaşımlarını seçme ve kullanma ihtiyacını karşılar. Tasarım sonuçlarının karşılaştırılması için farklı zorluk seviyelerinde en yaygın kullanılan makine elemanları ders kitapları, ulusal ve uluslararası standartları içeren beş farklı tasarım yaklaşımı seçildi. Her yaklaşımın sonuçları bir sonlu elemanları methodu, ANSYS kullanılarak analiz edildi. Ve her bir yaklaşımdan elde edilen kayıp ya da kazanç belirlendi ve sonuçlar dişli bozulma kriterleri, hız oranları ve güç aktarma aralıkları göz önünde bulundurularak karşılaştırmalı olarak verildi. Uygun tasarım yaklaşımını seçmek için faydalı çıktılar, pratik eğriler ve çizelgeler sunuldu. Buna ilaveten, çalışma, basit dişli tasarım yaklaşımlarının sonuçlarının ya da beş farklı yaklaşımdan seçilen herhangi birinin ANSI/AGMA Standardına çarpmak için kullanılabilen dönüşüm faktörleri sağlar. Ayrıca dişli tasarımını optimize etmeyi amaçlayan tasarımcılar ve öğrenciler için en iyi yaklaşımı önerir.

Anahtar Kelimeler: Düz dişli, Tasarım yaklaşımları, Tasarım çıktıları, Karşılaştırma

ACKNOWLEDGEMENT

Foremost, I would like to express my sincere gratitude to my supervisor Prof. Dr. Necdet GEREN for his invaluable guidance, advices, encouragement to develop myself and my studies. Feeling his support, both personal and professional helped me a lot during my studies.

I would like to thank specially to Prof. Dr. Hüseyin AKILLI for his continuous morale and encouraging advices from my undergraduate education.

Many thanks to my friends, Ayşe Nur ACAR and Hulusi DELİBAŞ for their help for finding tools about my study.

Lastly and the most importantly, I would like to thank my father İrfan UZAY and my mother Halime UZAY for their endless support, patience, encouragement and trust.

CONTENTS **PAGE**

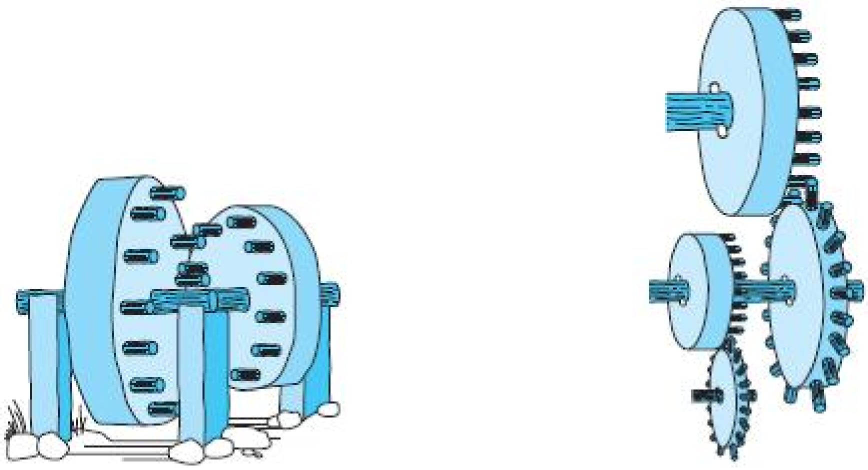
LIST OF FIGURES **PAGE**

LIST OF TABLES **PAGE**

1. INTRODUCTION

1.1. History of Gears

Gears, defined as toothed members transmitting rotary motion from one shaft to another, are among the oldest devices and inventions of man. In about 2600 B.C. the Chinese are known to have used a chariot incorporating a complex series of gears like those illustrated in Figure 1.1. Aristotle, in the fourth century B.C., wrote of gears as though they were commonplace. In the fifteenth century A.D., Leonardo da Vinci designed a multitude of devices incorporating many kinds of gears. Among the various means of mechanical power transmission (including primarily gears, belts, and chains), gears are generally the most rugged and durable. Their power transmission efficiency is as high as 98 percent (Juvinall R.C. and Marshek K.M., 2011).



Right angle gearing Parallel gearing

Figure 1.1. Primitive Gears

Ancient engineers were designing custom gears (Figure 1.2) for particular applications based on the knowledge of desired performance (input and output parameters) and available power sources, such as gravity, water current, wind, spring force, human or animal muscular power, etc. This knowledge allowed them to define gear arrangement and geometry, including a number of stages, location and rotation directions of input and output shafts, shape and size of the gear wheels, profile and number of teeth, and other parameters. Gear design also included material selection, which should provide the required strength and durability of every component in the gear drive (Alexander L. et al., 2013).

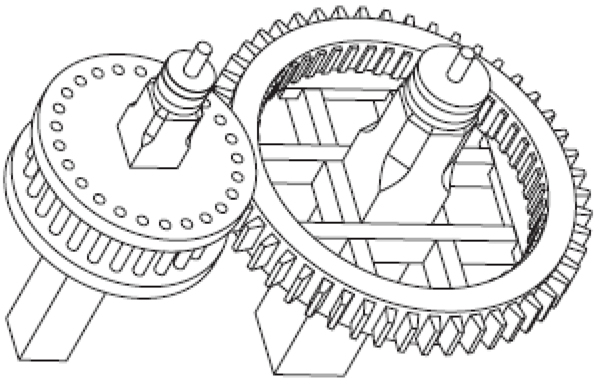


Figure 1.2. Ancient gear drive

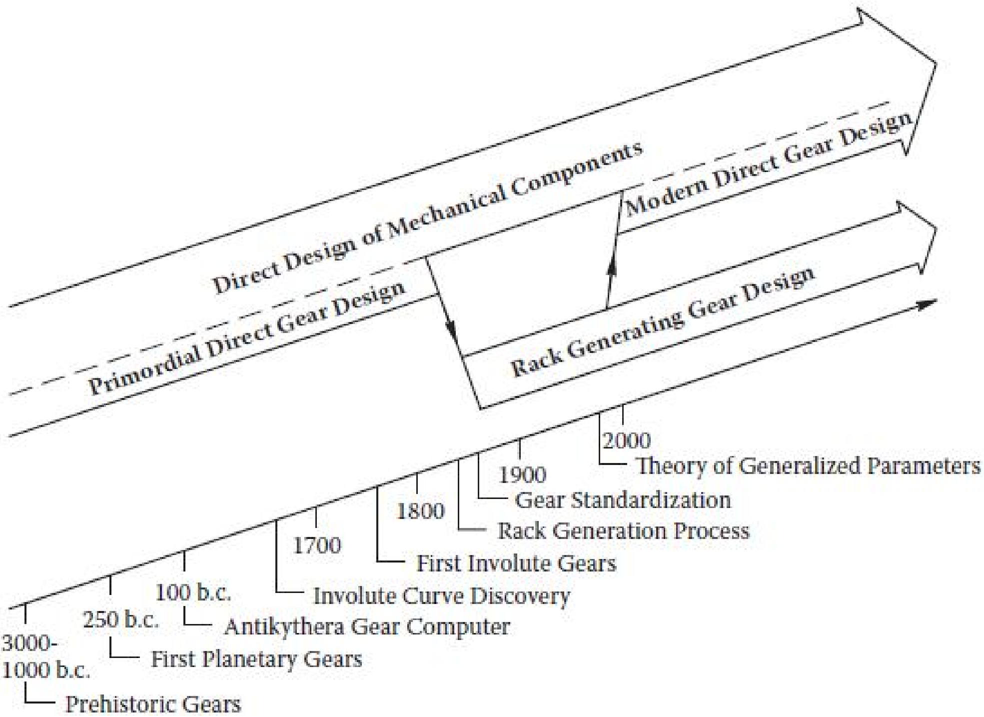


Figure 1.3. Evolution of Direct Gear Design (Alexander L. et al., 2013)

1.2 Gear Transmission

For transmissions where compact size, high efficiency or high speed are required, gears offer a competitive solution in comparison to other types of drive, such as belts and chains.

Gears are used in nearly all applications where power transfer is required, such as automobiles, industrial equipment, airplanes, helicopters, and marine vessels. A gearbox as usually used in the transmission system is also called a speed reducer, gear head, gear reducer etc., which consists of a set of gears, shafts and bearings that are factory mounted in an enclosed lubricated housing. Speed reducers are available in a broad range of sizes, capacities and speed ratios. Their job is to convert the input provided by a prime mover, usually an electric motor, into an output with lower speed and correspondingly higher torque.

Gear transmissions are widely used in various industries and their efficiency and reliability are critical in the final product performance evaluation. Gear transmissions affect energy consumption during usage, vibrations, noise, and warranty costs among others factors. These factors are critical in modern competitive manufacturing, especially in the aviation industry which demands exceptional operational requirements concerning high reliability and strength, low weight and energy consumption, low vibrations and noise. Considering their reliability and efficiency are some of the most important factors, problems of distribution of loads and, consequently, distribution of stresses in the whole gear transmission, particularly in teeth of mating gears, need to be thoroughly analyzed (Kawalec A. et al., 2006).

In this study, an involute spur gear design has been performed at speed ratios of 1:1, 3:1, 5:1, 8:1, and 10:1. And these speed reductions has been carried out at different amount of power transmissions of 1 kW, 10 kW, 100 kW, 500 kW and 1000kW.

1.3. Conjugate Action

The basic law of conjugate gear tooth action states that as the gears rotate, the common normal to the surfaces at the point of contact must always intersect the line of centers at the same point P, called the pitch point. The law of conjugate gear tooth action can be satisfied by various tooth shapes (Juvinall R.C. and Marshek K.M., 2011).

In theory, at least, it is possible arbitrarily to select any profile for one tooth and then to find a profile for the meshing tooth that will give conjugate action. One of these solutions is the involute profile, which, with few exceptions, is in universal use for gear teeth. When one curved surface pushes against another, the point of contact occurs where the two surfaces are tangent to each other and the forces at any instant are directed along the common normal to the two curves. To transmit motion at a constant angular velocity ratio, the pitch point must remain fixed; that is, all the lines of action for every instantaneous point of contact must pass through the same point P. In the case of the involute profile, it will be shown that all points of contact occur on the same straight line, that all normal to the tooth profiles at the point of contact coincide with the line, and, thus, that these profiles transmit uniform rotary motion (Budynas R.G. and Nisbett J.K., 2011).

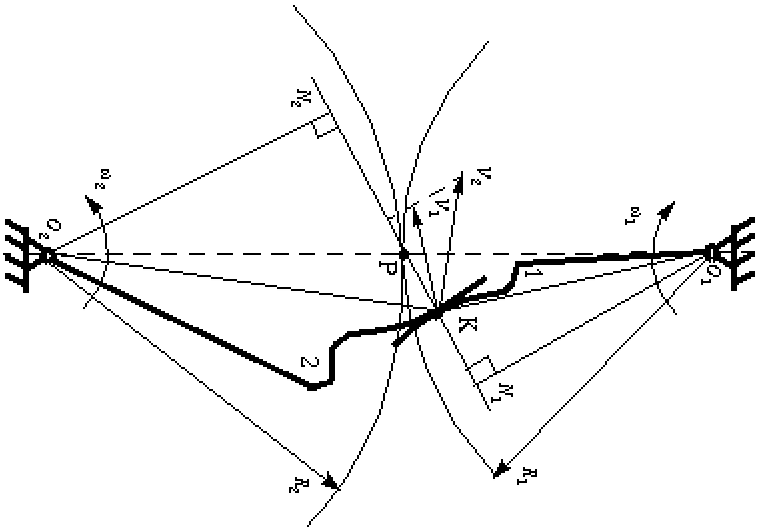


Figure 1.4. Conjugate Gear Tooth Action

1.4. Involute Profile

Simple teeth on a cylindrical wheel have some disadvantages that the speed ratio is not constant and the speed reduction causes noise and vibration problems especially at elevated speeds while a pair of gear is in a mesh. For this purpose, different kinds of geometrical forms can be used but the full depth involute profile is currently used in most engineering practices.

An involute of the circle is the curve generated by any point on a taut thread as it unwinds from a circle, called the base circle. The generation of two involutes is shown in Figure 1.5. The dotted lines show how these could correspond to the outer portions of the right sides of adjacent gear teeth. Correspondingly, involutes generated by unwinding a thread wrapped counterclockwise around the base circle would form the outer portions of the left sides of the teeth. Note that at every point, the involute is perpendicular to the taut thread (Juvinall R.C. and Marshek K.M., 2011).

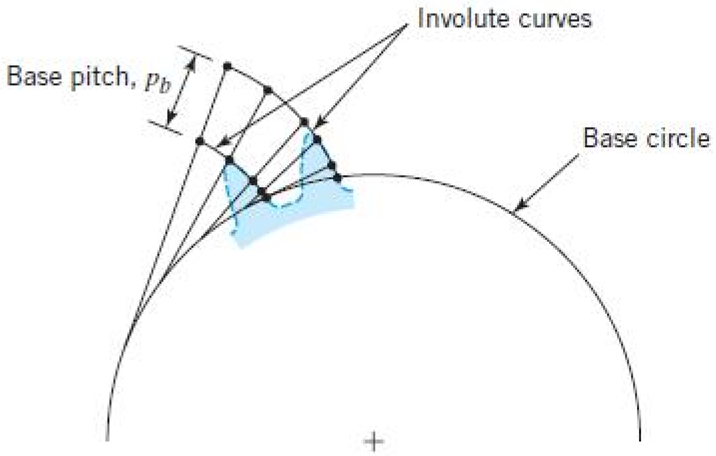


Figure 1.5. Generation of an involute from its base circle (Juvinall R.C. and Marshek K.M., 2011)

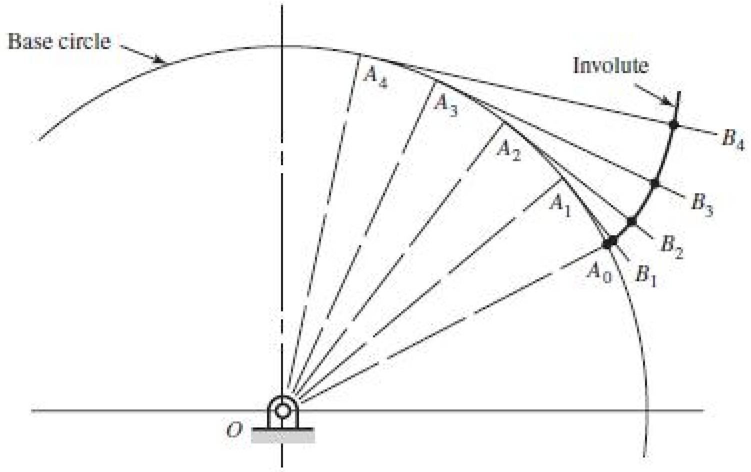


Figure 1.6. Construction of an involute curve (Budynas R.G. and Nisbett J.K., 2011)

As shown in Figure 1.6, divide the base circle into a number of equal parts, and construct radial lines OA_0, OA_1, OA_2, etc. Beginning at A_1, construct perpendiculars A_1B_1, A_2B_2, A_3B_3, etc. Then along A_1B_1 lay off the distance A_1A_0, along A_2B_2 lay off twice the distance A_1A_0, etc., producing points through which the involute curve can be constructed (Budynas R.G. and Nisbett J.K., 2011).

1.5. Gear Classification

Gears can be divided into a several classifications based on the arrangement of the axes of the gear pair and generally categorized as spur gears, helical gears, bevel gears and worm gears.

1.5.1. Spur Gears

The spur gear has teeth on the outside of a cylinder and the teeth are parallel to the axis of the cylinder. This simple type of gear is the most common and most used type. Spur gears are ordinarily thought of slow-speed gears, while helical gears are thought of as high-speed gears. If noise is not a serious design problem, spur gears can be used at almost any speeds that can be handled by other types of gears. Aircraft gas-turbine spur gears sometimes run at pitch-line speeds above 50 m/s (10,000 fpm). In general, though, spur gears are not used much above 20 m/s (4000 fpm).

Spur gear teeth may be hobbed, shaped, milled, stamped, drawn, sintered, cast, or shear cut. They may be given a finishing operation such as grinding, shaving, lapping, rolling, or burnishing. Speaking generally, there are more kinds of machine tools and processes available to make spur gears than to make any other gear type. This favorable situation often makes spur gears the choice where manufacturing cost is a major factor in the gear design.

The shape of the tooth is that of an involute form. There are, however, some notable exceptions. Precision mechanical clocks very often use cycloidal teeth since they have lower separating loads and generally operate more smoothly than involute gears and have fewer tendencies to bind. The cycloidal form is not used for power gearing because such gears are difficult to manufacture, sensitive to small changes in center distance, and not as strong or as durable as their involute brothers (Stephen P. R., 2012). In addition to that cycloidal teeth were in general use in the nineteenth century because they were easier to cast than involute teeth (Budynas R.G. and Nisbett J.K., 2011).

The standard measure of spur gear tooth size in the metric system is the module. The module of a gear plays an important role for power transmission between two shafts. The spur gear with a higher module is preferable for transmitting large amount of power between the parallel shafts. As a general rule, spur gears have a face width (F) from 3 to 5 times the circular pitch (p). As a result of this, a proper module selection and good combination of face width becomes the most important design parameters for the design and analysis of all gears including the spur gears.

1.5.2. Helical Gears

Helical gears have teeth inclined to the axis of rotation and transmit motion between parallel axes but sometimes this type of gears can be used for transmission between non-parallel shafts. Helical gears are typically used for heavy-duty high speed (>3500 rpm) power transmission, turbine drives, locomotive gearboxes and machine tool drives. Helical gears are generally more expensive than spur gears. Noise levels are lower than for spur gears because helical teeth enter the meshing zone progressively and make point contact in mesh rather than line contact and, therefore, have a smoother action than spur gear teeth and tend to be quieter.

In addition, the load transmitted may be somewhat larger, or the life of the gears may be greater for the same loading, than with an equivalent pair of spur gears. Conversely, in some cases, smaller helical gears (compared with spur gears) may be used to transmit the same loading. Helical gears produce an end thrust along the axis of the shafts in addition to separating and tangential (driving) loads of spur gears. Where suitable means can be provided to take this thrust, such as thrust collars or ball or tapered roller bearings, it is no great disadvantage. The efficiency of a helical gear set, which is dependent on the total normal tooth load (as well as the sliding velocity and friction coefficient, etc.), will usually be slightly lower than for an equivalent spur gear set (Stephen P. R., 2012).

1.5.3. Bevel Gears

Bevel gears have teeth formed on conical surfaces and are used mostly for transmitting motion between intersecting shafts. Bevel gears are used for motor transmission differential drives, valve control and mechanical instruments. A variety of tooth forms are possible, including straight bevel gears, spiral bevel gears, and zerol bevel gears. Straight bevel gears have a straight tooth form cut parallel to the cone axis, which if extended would pass through a point of intersection on the shaft axis. Straight bevel gears are usually only suitable for speeds up to 5 m/s. Spiral bevel gears have curved teeth that are formed along a spiral angle to the cone axis. The advantage of spiral bevel gears over straight teeth is that the gears engage more gradually, with contact commencing at one end of the tooth which increases until there is contact across the whole length of the tooth. This enables a smoother transmission of power and reduces the risk of tooth breakage. Spiral bevel gears are recommended for pitch line speeds in the range from 5 to 40 m/s. Zerol bevel gears have a tooth form that is curved but with a zero spiral angle. They represent an intermediate category between straight and spiral bevel gears (Childs. Peter R. N., 2013).

1.5.4. Worm Gears

A worm gear is a cylindrical helical gear with one or more threads and resembles a screw thread. A worm wheel is a cylindrical gear with flanks cut in such a way as to ensure contact with the flanks of the worm gear. Worm gears are used for steering gear, winch blocks, low speed gearboxes, rotary tables and remote valve control. Worm gear sets are capable of high-speed reduction and high load applications, where non-parallel, non-interacting shafts are used. Heat generation due to friction is high in worm gears, for this reason a lubrication process is needed continuously. Worm and wheel gears are widely used for non-parallel, non-intersecting, right angle gear drive system applications where a high transmission gearing ratio is required. In comparison to other gear, belt, and chain transmission elements, worm and wheel gear sets tend to offer a more compact solution. Worm and wheel gear sets can achieve gear ratios of up to 360:1 compared to other gear sets, which are typically limited to a gear ratio of up to 10:1 (Childs. Peter R. N., 2013).

1.6. Aim of Study

Gears are always subjected to bending and surface contact stresses under working conditions by applied load or torque during the transmission of power. Bending stress occurs at the root of the tooth profile mainly, and surface contact stress occurs in the gear tooth surface while a pair of gear is transmitting power. Bending stress is the highest at the fillet and can cause breakage or fatigue failure of tooth in root region. Whereas surface contact stresses are on the side of tooth may causes scoring wear, pitting fatigue failure.

The best combination of two design parameters that are module (m) and face width (F) are searched in the gear design, if material is pre-selected. After defining the pinion and gear materials, module is estimated and calculations are carried out to determine the face width. Module and face width calculations are iterated until the face width is in a range of $3p \leq F \leq 5p$ where p (p.m) is circular pitch that is dependent on the selected module. The iteration may require considerable time depending on the initially selected module, which is dependent on expertise.

Various design formulas are available in the machine elements or machine design text books for the design or finding “m” or “F”. In each kind of approaches, the effect of some factors are more dominant than others. In addition to this, the international and national standards such as American Gear Manufacturers Association (ANSI/AGMA), American Petroleum Institute (API), Deutsches Institut für Normung (DIN), Japanese Gear Manufacturers Association (JGMA) and International Organization for Standardization (ISO) provide different formulae with different level of difficulty.

However, the results of using different approaches have not been compared so far. Thus the designer does not aware of the success or loss gained using each of the approach. Therefore, there is a need to compare the results of each of the most accepted design formula or design approach for the involute spur gear design.

In this study module selection and face width calculations have been carried out based on bending stress and surface contact stress using the design approaches provides in the most accepted reference books and standards such as Mechanical Engineering Design 1st Metric Edition (Shigley J.E., 1985), Shigley's Mechanical Engineering Design 9th Edition (Budynas R.G. and Nisbett J.K., 2011), Fundamentals of Machine Component Design 5th edition (Juvinall R.C. and Marshek K.M., 2011), ISO 9085:2002 Standards (2002), ISO 6336 Standards (2006) and ANSI/AGMA 2101-D04 Standards (2004).

The main intention is to compare the design results given by the most commonly used gear design approaches. Hence, the designer can be aware of the success or loss gained using each of the approach. The results of the study may also help to select the proper gear design approach depending on the requirements of the particular design.

2. PREVIOUS STUDIES

Various studies are available for the design of an involute spur gear in literature. Almost all works are related to decrease bending and surface contact stresses. In order to decrease these stresses, researches put efforts improving gear profile and optimization of dimensions by using different kind of methods mentioned in below sections.

2.1 Most Common Gear Design Approaches

Design of an involute spur gear requires number of determinations that including different design factors. In order to perform a spur gear design, national and international standards and/or machine elements textbooks have been provided for designer. In this study different machine element textbooks have been searched and three of them (Shigley's Mechanical Engineering Design 1st Metric Edition and 9th Edition, Fundamentals of Machine Component Design 5th Edition) have been considered which are most commonly used and introducing a design of spur gear clearly. In addition to these textbooks ANSI/AGMA Standards and ISO Standards have been studied also. Since the some kind of textbooks have shown similar procedure with ANSI/AGMA or ISO Standards they have been eliminated, see Table 2.1.

Table 2.1. Literature search that related to design of spur gear design

Available Design Approaches	**The main Basis of Design approach**
ANSI/AGMA Standards	ANSI/AGMA Standards*
ISO Standards	ISO Standards*
DIN Standards	ISO Standards
Mechanical Engineering Design 1st Metric Edition (Shigley's J.E., 1985)	ANSI/AGMA Standards**
Shigley's Mechanical Engineering Design 9th Edition (Budynas R.G. and Nisbett J.K., 2011)	Lewis and Hertzian Theory and includes ANSI/AGMA Standards*
Fundamentals of Machine Component Design 5th Edition (Juvinall R.C., Marshek K.M., 2011)	Similar to ANSI/AGMA**
Mechanical Design Engineering Handbook 1st Edition (Childs P.R.N., 2013)	ANSI/AGMA Standards
Mechanical Design: An Integrated Approach (Ugural A.C., 2003)	ANSI/AGMA and/or Fundamental of Machine Component Design
Gears and Gear Drives (Jelaska D.T, 2012)	Combination of the ISO 6336 and DIN 3990 Standards
Machine Elements in Mechanical Design (Mott R.L., 2003)	ANSI/AGMA Standards
Makine Elemanları ve Konstrüksiyon Örnekleri (Babalık F.C., 2010)	DIN Standards

*Most commonly used
** Introduces the design of a spur gear clearly

2.2. Gear Design using Computrer Aided Engineering (CAE)

The term computer aided engineering (CAE) generally applies to all computer related engineering applications (Budynas R.G. and Nisbett J.K., 2011).

The CAE systems make sophisticated mathematical algorithms to perform calculations. To carry out the simulation calculations, CAE uses finite technical elements. The working procedures to carry out simulations with CAE systems can be divided in three main phases: pre-processing (generation of the sweater of finite elements and variables of entrance), processing (calculation of the demands) and post-processing (evaluation and interpretation of the answer of the software) (Gökçek M., 2012).

There are different software's available for modeling. Some of them are Aries, AutoCAD, Cad Key, Solid works, Pro Engineer, I-Deas, Inventor, Mechanical desktop, Unigraphics, Catia V5 and etc.

The finite element method (FEM) is numerical analysis technique for obtaining approximate solutions to a wide variety of engineering problems. Because of its diversity and flexibility as an analysis tool, it is receiving much attention in almost every industry. Since it is not possible to obtain analytical mathematical solutions for many engineering problems, it is necessary to obtain approximate solutions to problem rather than exact closed form solution. The finite element method has become a powerful tool for the numerical solutions of a wide range of engineering problems. Various software's are available for finite element analysis (FEA) such as Altair hyper works, Ansys, Nastran, Cosmos, LS-Dyna (Parthiban A.et al, 2013).

Geren N. and Baysal M. (2000) developed an expert system which is a branch of Artificial Intelligence. They used this system for gearbox design by operating Delphi from Borland for an expert system development tool. And the American Gear Manufacturers Association (AGMA) methods and its recommendations were used for designing the spur gear. The developped programme by Geren N. and Baysal M., has a user friendly interface that allows to dealer to select the type of gear, material and etc. The program also includes recommended module size list box which is the result of estimating gear size procedure. This program has advantages of decreasing the design duration to 2 minutes for experienced designer and few minutes for inexperienced designer, allowing the user to try different design alternatives in a short time, eliminating the errors made during the manual design process, warning and directing the user to go on proper design, having an expandable database, reducing the design cost for each gearbox and behaving as a tutorial.

A batch module called “integration of finite element analysis and optimum” was established by using combination of I-DEAS, ABAQUS/Standard and MOST softwares by Li C. et al (2002). The geometrical model, contact stress analysis and finding the optimal solution were carried out automatically by this batch module with the given input variables. In order to validate the usage of this integrated module, two gear systems were selected as a testing examples, a pair of pinion and gear and a planetary gear system. Optimum solution considering the gear stress was found as 302,5 MPa at a pressure angle of $23{,}6^{\circ}$ after 85 iterations for a pinion and gear system. And for the planetary gear system the stress was found as 330,24 MPa when the inner radius of the planetary gear was 5,1 mm. They concluded that this kind of study provides a high efficiency for gear design procedure that saves time and resources.

Gologlu C. and Zeyveli M. (2009) applied genetic algorithm (GA) to the design of helical gear trains problem. It was aimed to minimize the volume of gears by using penalty functions that depend on design variables and its constraints. The time duration for a design of helical gear trains was 4 seconds but on the other hand the face width of gears was not found in a range of (3p, 5p).

Mendi F. et al. (2010) developed Borland Delphi 6.0 platform to execute GA. They performed the dimensional optimization of motion and force transmitting components of a gearbox by GA. Selection of optimum module was carried out using GA. When results were compared to analytical results GA has given better results such as lower module, less volume. But the face width that calculated by GA was out of limits determined by 3 to 5 times of circular pitch.

Huang K. J. and Su H. W., (2010) investigated to construct mesh elements and dynamic analysis of spur/helical gears that includes cylindrical and conical categories with consisting of modifications such as tip relief, crowning modification, and undercutting, see Figure 2.1. And it is seen form Figures 2.2 and 2.3 meshed elements were constructed directly with mathematical profile equations of the gears via a C code. By using the Newton–Raphson method, the coordinates of intersection point among nonlinear tooth profiles were obtained. Finally, dynamic responses of spur and helical gear pairs were calculated by LS-DYNA. This type of approach was decided as it is adequate to wide categories of dynamic gear problems under sophisticate design considerations.

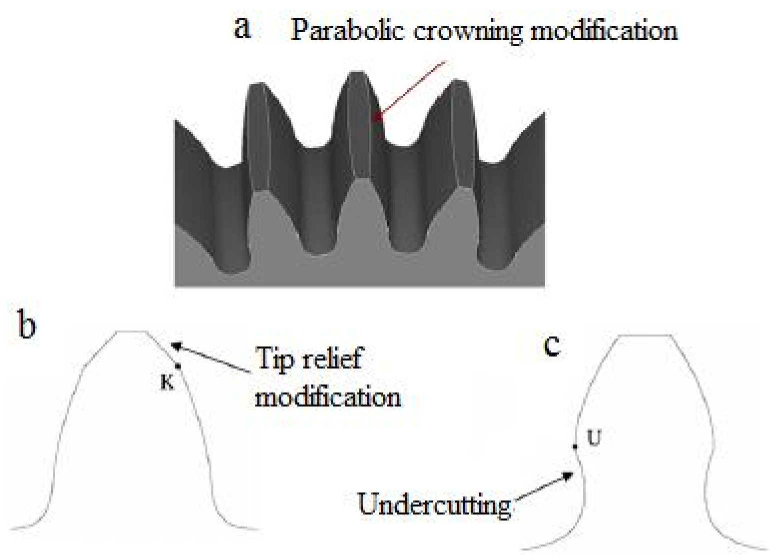


Figure 2.1. Modifications and undercutting in gears: (a) parabolic crowning modification, (b) tip relief modification, and (c) undercutting (Huang K. J. and Su H. W., 2010)

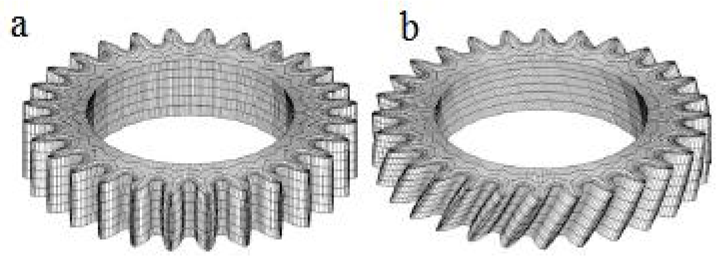


Figure 2.2. Elements of gears with tip relief (b_j=0.2mn, hj=0.3mn) and crowning modification factor (C_c=0.04) for (a) spur gear and (b) helical gear (Huang K. J. and Su H. W., 2010)

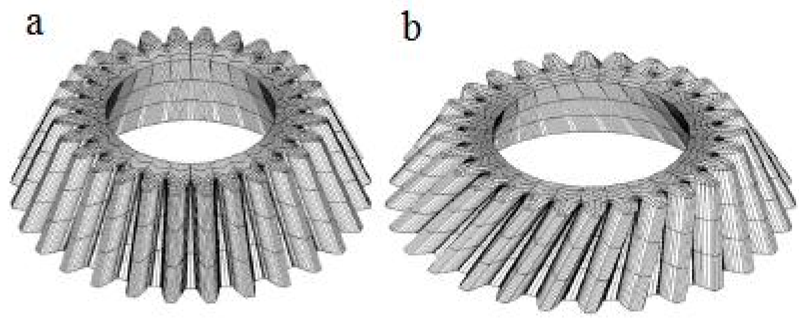


Figure 2.3. Elements of conical gears with conical pitch angle (g=30°), for (a) straight conical gear and (b) helical conical gear (Huang K. J. and Su H. W., 2010)

Kamble A. G. et al. (2010) used C++ language and found the dimensions of gear. If the material strength could be less than the calculated stress, the program could fail and the new iteration steps could be started by the user. After the program gave the results, mathematical models were developped to obtain more realistic results. And these results were verified by experimental methods to introduce optimum solution.

Parthiban A.et al, (2013) used Pro-E as modeling tool and examined the the tooth failure in spur gears. They investigated the optimization of gear profile geometry by using CAD &CAE and improving the gear tooth strength. Circular root was introduced instead of standard trochoidal root fillet in spur gear and analyzed by using CAE software. A spur gear had circular fillet with 15 teeth at 1000 rpm and 1500 rpm was analyzed and results were shown in Tables 2.2 and 2.3. It was concluded that increasing speed has resulted less deformation and less von Misses stress, and the circular root fillet design was particularly suitable for lesser number of pinions and whereas the trochoidal root fillet was more appropriate for higher number of teeth.

Table 2.2. Circular fillet 15 teeth 1000 rpm (Parthiban A.et al, 2013)

ALLOY STRUCTURAL STEEL	MIN	MAX
Total defromation (mm)	0	0,043335
Equivalent Elastic Stress (MPa)	0,32579	45,043

Table 2.3. Circular fillet 15 teeth 1500 rpm (Parthiban A.et al, 2013)

ALLOY STRUCTURAL STEEL	MIN	MAX
Total defromation (mm)	0	0,0028892
Equivalent Elastic Stress (MPa)	0,2194	30,041

2.3 Verification of Gear Design Results with Finite Element Analysis

Gear design is performed considering to the fatigue bending stress and contact stress. The tooth root is subjected to fatigue bending stress and the tooth surfaces are subjected to fatigue contact stresses. Determination of gear stresses have significant importance because failure of a gear due to bending causes tooth breakage whereas due to surface contact causes pitting, scoring and/or wear.

Tiwari S. K. et al.(2012), Karaveer V. et al. (2013), Shinde S.P. et al. (2009), Ambade V.V. et al (2013) and Fetvacı M.C. et al.(2004) have analyzed gear stresses by using a FEM and compared with theoretical calculations such as Lewis formula, Hertzian equation and AGMA standards. They have concluded that FEM is in a good agreement with analytical approaches. Tiwari S. K. et al.(2012) showed the results in Table 2.4. and 2.5, Karaveer V. et al. (2013) gave the results by showing the difference between analytical and numerical approaches in Table 2.6. But Ambade V.V. et al. (2013) have compared the results of equivalent stresses by FEA with theoretical approaches. And decided to make a further modification to validate the results. And Fetvacı M.C. et al. (2004) have indicated that the root area affected by applied boundary conditions by using gear with one tooth model does not give appropriate result and suggested to use fully isolated root area for gear model.

Table 2.4. Comparison of root bending stress results (Tiwari S. K. et al., 2012)

ROOT BENDING STRESS		
	Pinion	Gear
Lewis Formula	63,00 MPa	46,39 MPa
AGMA bending stress	60,39 MPa	48,18 MPa
FEA stress	55,61 MPa	42,94 MPa
FEA (in meshing) Maximum Principle stress	59,73 MPa	

Table 2.5. Comparison of contact stress results (Tiwari S. K. et al., 2012)

CONTACT STRESS	
Hertz Equation	-562,27 MPa
AGMA contact stress	572,00 MPa
FEA	567,75 MPa

Table 2.6. Comparison of maximum contact stress obtained from Hertz equation and ANSYS 14.5 (Karaveer V. et al., 2013)

Gear	s_a **(Hertz) (MPa)**	s_a **(ANSYS) (MPa)**	**Difference (%)**
Steel	2254,9821	2261,2052	0,28
Grey CI	2334,6414	2365,1782	1,29

Gupta B. et al. (2012) have studied contact stress analysis with different module of spur gear using finite element analysis. The contact stress was compared with Hertzian stresses obtained using analytical approach given by Hertzian equation. It was showed that the module is important geometrical parameter during the design of gear. They concluded that if the contact stress minimization is the primary concern and if the large power is to be transmitted then a spur gear with higher module is preferred. Because of reduction in contact stress as it is seen in Table 2.7.

Table 2.7. Comparison of peak values of the contact stresses by considering different modules (Gupta B. et al., 2012)

Sr. No.	Module (mm)	$P_{p(ANSYS)}$ (MPa)	$P_{p(Hertzian\ stress)}$ (MPa)	Differences (%)
1	2	1733,7	1724,13	0,5
2	3	800,6	791,02	1,19
3	4	468,64	465,56	0,65
4	5	255,43	257,34	0,74
5	7	129,83	129,61	0,16
6	8	102,41	102,85	0,43
7	9	53,457	52,39	1,97

Jebur A.K. et al. (2013) have investigated the contact stresses between pair of the gears (surface to surface) by using ANSYS software and compared the results with the experimental results which was established by using the D.C servomotor and planting the strain gages in the tooth of the gear. This research showed the difference between numerical and experimental approaches that were very small and equal to 12,86 %. FEA model (surface to surface) could be used to simulate contact between two bodies accurately by verification of contact stresses between two gears in contact. It has been underlined that module has the greater effect on the behavior of the tooth contact stresses. Because decreasing the module leads to increase in the contact stress. And also they concluded that increasing the spur gear design parameters (number of teeth with module) leads to improvement in the tooth strength. Since the thickness of the critical section will increase, gear tooth withstands higher loads.

Sanchez M. et. al. (2013) used a non-uniform model of load distribution along the line of contact of spur and helical gears and analyzed the critical value of the stress and the critical load conditions by using minimum elastic potential criterion and a complete analysis of the tooth bending strength was carried out. The elastic potential energy of a pair of teeth was calculated and expressed as a function of the contact point and the normal load. This approach allowed to know the value of the load per unit of length at any point of the line of contact and at any position of the meshing cycle. To validate the bending strength model a study by the FEM was carried out. Several cases were considered with different number of teeth on pinion and wheel, and different gear ratios, including spur and helical gear teeth. When analyzing the tooth root stress, FEM stress was little lower than minimum elastic potential (MEP) model and the load sharing ratio for both FEM and MEP models almost same for spur gears. For helical gear, MEP bending stress was lower than FEM stress and MEP load values fitted quite accurately with FEM load values.

2.4. The studies on the Effect of Profile Modification

Since the gear stresses have to be taken into consideration for a design, various investigations on the tooth profile have been done in order to reduce these stresses.

Li S. (2008) used face-contact model of teeth, mathematical programming method (MPM) and three-dimensional (3D) FEM together to conduct loaded tooth contact analyses (LTCA), deformation and stress calculations of spur gears with different addendums and contact ratios. And as shown in Figure 2.4 the work in the paper investigated effects of addendum and contact ratio on tooth contact strength, bending strength and basic performance parameters of spur gears. Also transmission error and mesh stiffness of the two pairs of spur gears were analyzed. These methods that mentioned above was proved by experiments and ISO standards and showed that these methods can perform exact analyses of tooth surface contact stresses and root bending stresses of spur gears with standard addendum.

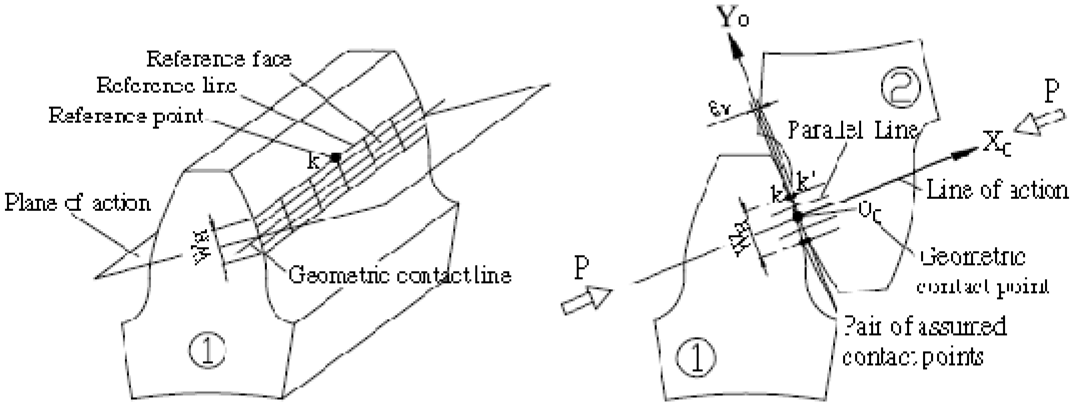


Figure 2.4. Face-contact model of gears (Li S., 2008)

Tooth contact stresses were calculated by dividing the tooth load obtained in LTCA with a contact area on the reference face as shown in Figure 2.4. Root bending stress along the tooth profile was calculated with the 3D, FEM. Figure 2.5 was the 3D, FEM model used for root bending stress analysis. (Elements: 54796, Nodes: 64140)

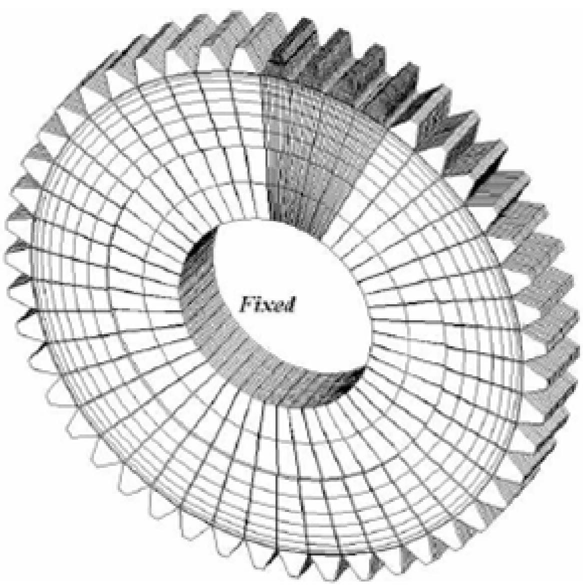


Figure 2.5. FEM model of the gear (Li S. 2008)

Li S. (2008) gave the following results;

a) Tooth load-sharing rate can be changed greatly if the number of contact teeth is increased through increasing addendum.

b) Tooth root stress is increased if addendum becomes longer and number of contact teeth has no change. Tooth root stress can also be reduced when the number of contact teeth is increased through increasing the addendum. But there is no guarantee that this increment of the number of contact teeth can certainly reduce the tooth root stress. This is because the increase of the addendum also makes the whole depth of teeth longer so that a larger moment occurred at the tooth root.
c) Tooth contact stress and contact width are changed slightly if addendum becomes longer and number of contact teeth is not changed. But they are reduced greatly if the number of contact teeth is increased through increasing the addendum.
d) Transmission error of the gears is increased if addendum becomes longer and number of contact teeth is not changed. But it can also be reduced by increasing the number of contact teeth.
e) Mesh stiffness is reduced if addendum becomes longer and number of contact teeth is not changed. But it can also be increased by increasing the number of contact teeth.

Pedersen N.L. (2010) has shown that bending stress could be reduced significantly by using asymmetric gear teeth and by shape optimizing of gear that changes made to the tool geometry. Root shape optimization has been achieved by designing a cutting tool with changing coast side and drive side pressure angles. The results were examined by three gears with different number of teeth and all teeth were cut with a rack tooth that have a height of 2,25m. And it was concluded that the largest reduction in the bending stress (44,3%) could be found if the drive side pressure angle is greater than coast side pressure angle (α_d=36° and α_c=20° and gear with 68 teeth). In Figure 2.7 the stress was reduced with 39,2% as compared to the ISO profile where the best design for a gear with 17 teeth gave a stress reduction of 12,2% compared to the ISO tooth.

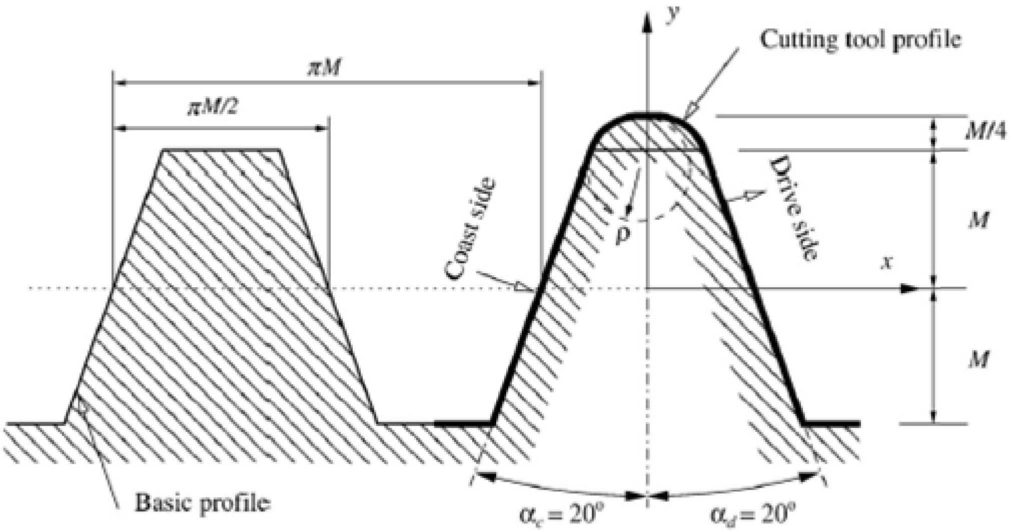


Figure 2.6. Cutting profile geometric definition and the basic profile based on the ISO profile (Pedersen N.L., 2010)

In Figure 2.6, M is the gear module that defines the teeth size in the gear. The two sides of the tool are termed drive and coast side respectively. Pressure angles α_d and α_c are shown here with the same value.

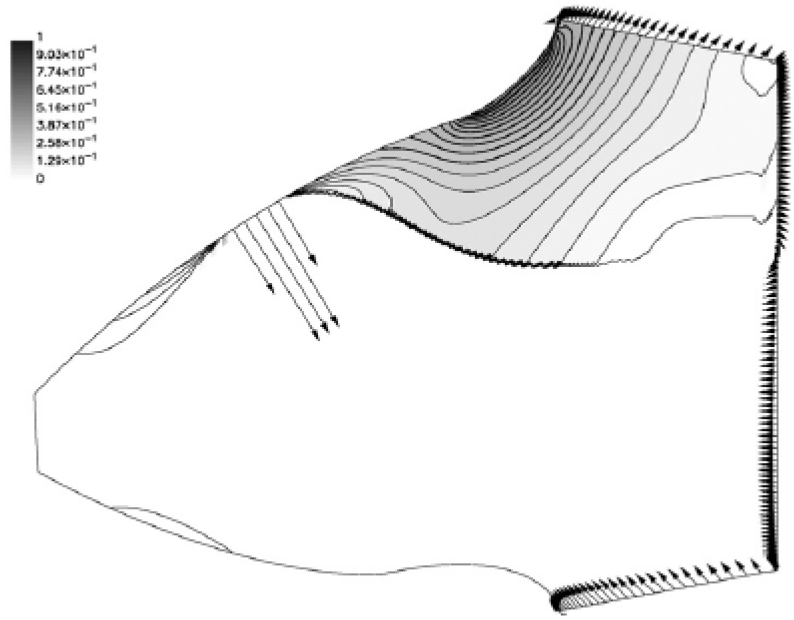


Figure 2.7. Tooth form (gear with 17 teeth and α_d=35° and α_c=20°) that used in FE modeling (Pedersen N.L., 2010)

Marković K. et al. (2011) have observed the linear tip relief profile modification. Tip relief profile modification was defined as the thickness Δ_s(d) of the material removed along the tooth flank with reference to the nominal involute profile. Tooth tip diameter d_a, profile relief at tooth tip C_a and diameter at the beginning of correction d_k had to be calculated to define changes in tooth thickness.

$$\Delta_s(d) = C_a \cdot \frac{d - d_k}{d_a - d_k}$$

The standard gear numerical model and modified gear were developed and analyzed by using the finite element method. And results were showed that there has been a stress decrease in the teeth flank at the tip area, and the same situation has appeared at the end of contact between meshing gears with linear tip relief profile modification.

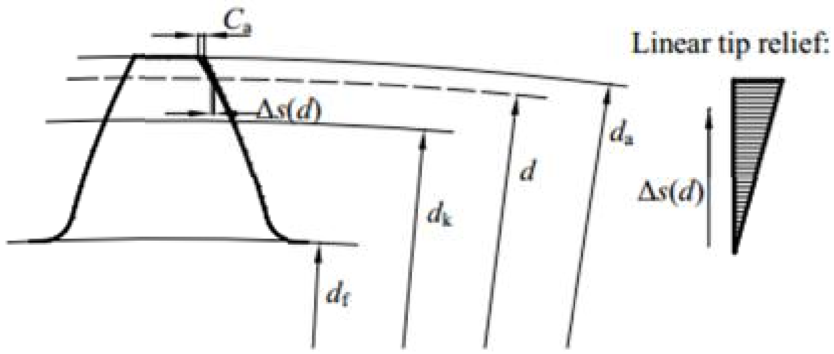


Figure 2.8. Linear tip relief profile modification (Marković K. et al., 2011)

Gurumani R. et al. (2011) studied effect of major performance characteristics of uncrowned spur gear teeth at the pitch point and compared with longitudinally modified spur gear teeth as it is seen in Tables 2.8 and 2.9. They presented the results of 3D FEM analyses using ANSYS. This study have concluded that the transmission error of spur gear which has large changes in mesh stiffness can be reduced by applying the proposed crowning modification.

Table 2.8. Contact parameters of crowned spur gear teeth (Gurumani R. et al., 2011)

Contact parameters	**Contact type**		
	Line contact	**Elliptical contact**	
		Circular crowned	**Involute crowned**
Semi contact width or contact radius, mm	0,1152	semi-major axis a=3,4225 semi-minor axis b=0,1152	semi-major axis a=3,4 semi-minor axis b=0,1152
Contact area, mm^2	6,912	1.239	Oca.23
Max. Contact pressure (Hertz stress),N/mm^2	1238	7792	7851
Approaches of centres (Normal displacement), mm	0,0073	0,0134	0,0135
Normal stiffness, N/mm	887,794	311,407	314,843

Table 2.9. Value of performance parameters of standard spur gear teeth by FEM (Gurumani R. et al., 2011)

Tooth modification	Contact stress, MPa	Tooth bending stress at root, MPa	Gear tooth deflection, mm
Non-modified	1021	393,38	0,0166
Circular crowned	6078	1025	0,0572
Involute crowned	5947	946	0,0577

Sankar S. et al. (2011) discussed about a novel method. The method uses circular root filet instead of standard trochoidal root filet, see Figure 2.9. It has been introduced in gears having less than 17 teeth to decrease the bending stress at the root and gear tooth failure due to undercutting. Stress analysis were made at different speeds for both circular and trochoidal root filet. It can be seen from Table 2.10 ANSYS results has indicated that the gears made of circular root filet has yielded better strength (reduced bending and contact shear stress) thereby improved the fatigue life of the gear material.

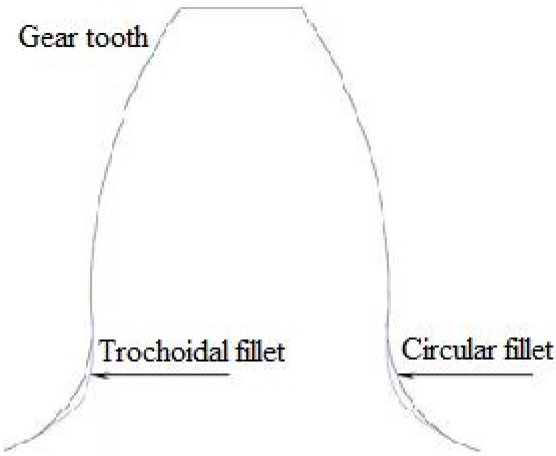


Figure 2.9. Superposition of circular filet on a standard tooth (Sankar S. et al., 2011)

Table 2.10. FEA results – contact shear stress (Sankar S. et al., 2011)

Speed (r/min)	Deflection (mm) for 13 teeth		Contact shear stress (N/mm^2)		Stiffness (N/mm) for 13 teeth	
	Trochoidal root fillet	Circular root fillet	Trochoidal root fillet	Circular root fillet	Trochoidal root fillet	Circular root fillet
1,500	0,019601	0,010894	451,594	412,089	186,332	335,258
2,000	0,014161	0,007904	326,113	299,253	193,418	346,533
2,500	0,014204	0,006527	287,021	247,243	192,480	335,714
3,000	0,010903	0,005441	250,904	206,085	203,046	335,627

Dhavale A.S. et al. (2013) have mentioned that due to the high stresses, possibility of fatigue failure at the root of teeth of spur gear increases and showed that these stresses could be minimized by introducing stress relief features at stress zone. They have generated holes to the tooth root area and compared the root fillet stresses with and without holes by using a FEM. The results showed that stress redistribution was highly sensitive to the change in size, location and number of holes. Using two holes as a stress reliving features has given more stress reduction.

2.5. Comparison of Gear Design Approaches

There are many gear tooth and gearbox rating standards existing in the world. For a given gearbox, the rating system that is used can give very different answers in the amount of power that can be transmitted. If a user is not specific or does not have a basic understanding of the different rating systems, the price and the reliability of the gearbox can be dramatically affected (Beckman K.O., Patel V.P., 2000).

National and international standards such as Japanese Gear Manufacturers Association (JGMA), American Gear Manufacturers Association (AGMA), American Petroleum Institute (API), Deutsches Institut für Normung (DIN), International Organization for Standardization (ISO) and various machine design textbooks provide different formulae with different level of difficulty.

Li S. (2006) used finite element analysis for contact strength and bending strength of a pair of spur gears with machining errors, assembly errors and tooth modifications. Rademacher’s experimental results were used to compare with the results obtained by the FEM. Surface contact stress and root bending stress of the same pair of spur gears were also calculated by Japanese Gear manufacturers Association (JGMA) and International Organization for Standardization (ISO) standards for comparing with the FEM results. This study concluded that when the effects of machining errors, assembly errors and tooth modifications are considered together at the same time, surface contact stress and root bending stress become much greater than the case without errors and tooth modifications.

Kawalec A. et al. (2006) have made comparative analysis of tooth root strength evaluation methods used within ISO and AGMA standards and verifying them with developed models and simulations using the FEM. They have concluded that tooth root stress obtained by FEM gave smaller values comparing to the calculations using ISO standard. In contrary to this, FEM gave greater values than the values carried out using AGMA standards. And also in the case of gears manufactured with racks, FEM stresses have been closer to ISO standards, in the case of gears manufactured with gear tool, FEM stresses have been closer to AGMA standards.

Kawalec A. et al. (2008) have indicated that there were no consistent procedures in the standards for cylindrical gears for computing correct geometric models of gears made with racks and gear tools. And they developed a suitable method for computation of parameters of critical section at tooth root of cylindrical gears, considering real and not virtual parameters of applied gear tool. This method has maintained the principles of ISO Standard. The developed method has allowed for using the ISO standard for tooth root strength of gears manufactured with gear tools, preserved its fundamental assumptions and advantages. They concluded that tooth root stresses were much closer to the results based on FEA and AGMA standard than to the ones obtained using the ISO standard.

Patel I. et al. (2013) have modeled spur gear in Pro engineer wildfire 5.0, then calculated the stresses on ANSYS workbench, and created a Simulink model using curve fitting equation. The results were compared with both AGMA and ANSYS. The results obtained from both ANSYS and Simulink were close to the results of AGMA which concluded that Simulink is also an equivalent tool if modeled properly by using curve fitting. Table 2.11 shows the bending stress results based on ANSYS, AGMA and Simulink approaches with different number of teeth on spur gear.

Table 2.11. Bending Stress in MPa (Patel I. et al., 2013)

S. No	No. of Teeth	ANSYS	AGMA	Simulink
1	15	10,77	10,017	10,49
2	16	10,49	10,68	10,95
3	17	11,53	11,35	11,46
4	18	12,23	12,02	12
5	19	12,38	12,68	12,59
6	20	13,445	13,35	13,22
7	21	13,4991	14,02	13,89
8	22	14,7	14,69	14,6
9	23	15,407	15,36	15,35
10	24	16,59	16,028	16,14
11	25	16,637	16,69	16,98

.

Hwanga S.C. et al. (2013) have performed contact stress analyses for spur and helical gears between two gear teeth at different contact positions during rotation. The variation of the contact stress during rotation has been compared with the contact stress at the lowest point of single tooth contact (LPSTC) and the AGMA equation for the contact stress. The change in the contact stress at any point of the line of contact has been analyzed through the finite element method. The maximum value of contact stress measured at the lowest point single-tooth contact has been compared with AGMA standard. According to the analysis and calculation results they have concluded that the FEM gear design that considers the contact stress was stricter than the AGMA standard.

2.6. Optimization Techniques used in Gear Designs

Marjanovic N. et al. (2012) presented the characteristics and problems of optimization of gear trains with spur gears. This study aimed to provide a description for selection of the optimal concept, based on selection matrix, selection of optimal materials, optimal gear ratio and optimal positions of shaft axes. Gear train optimization software had been used by a C++ language in order to reduce the volume of gear train and obtained a reduction by 22% in a very short time.

Golabi S. et al. (2013) investigated the general form of objective function and design constraints for the volume/weight of a gearbox by choosing different values for the input power, gear ratio and hardness of gears. Selected values for input parameters for gearbox optimization were given in Table 2.12. From the results, all the necessary parameters of the gearbox such as number of stages, modules, face width of gears, and shaft diameter were introduced. One, two and three-stage gear trains had been considered and by using a Matlab program, the volume/weight of the gearbox was minimized.

Table 2.12. Selected value of input data (Golabi S. et al., 2013)

Input parameters	Selected values
Transmission power	2, 5, 10, 20, 30, 50, 80, 100, 150, 200 hp
Hardness of material	200, 300, 400 BHN
Gearbox ratio	1.5, 2, 3, 5, 8, 10, 15, 20, 40, 50

2.7. Dimensionless Solution for Optimal Gear Design

Gear design can be optimized by making the parts with minimum size, optimal tooth geometry and selecting the proper materials which have a good physical property. Carroll R.K. and Johnson G.E., (1989), introduced a new dimensionless quantity called the Material Properties Relationship Factor, C_{MP}. They defined dimensionless space as the optimal gear geometry can be found independently of the load and speed requirements of the gear set. And for a given set of standard gear tooth parameters (pressure angle, addendum and dedendum ratios, face width to diameter ratio, etc.), the optimal gear set geometry depended only on the gear ratio, m_G, and the physical properties of the materials used. The advantage of this method provides a quick and sure approach.

Dimensionless number, C_{MP}, was introduced by a simplified equation as follow and the values of C_{MP} were showed in below figure and table based on gear stresses.

$$C_{MP}=\left(s_t.C_p{}^2/s_C{}^2\right)^{1/3}$$

Where

s_t: bending stress number,

s_c: contact stress number,

C_p: elastic coefficient

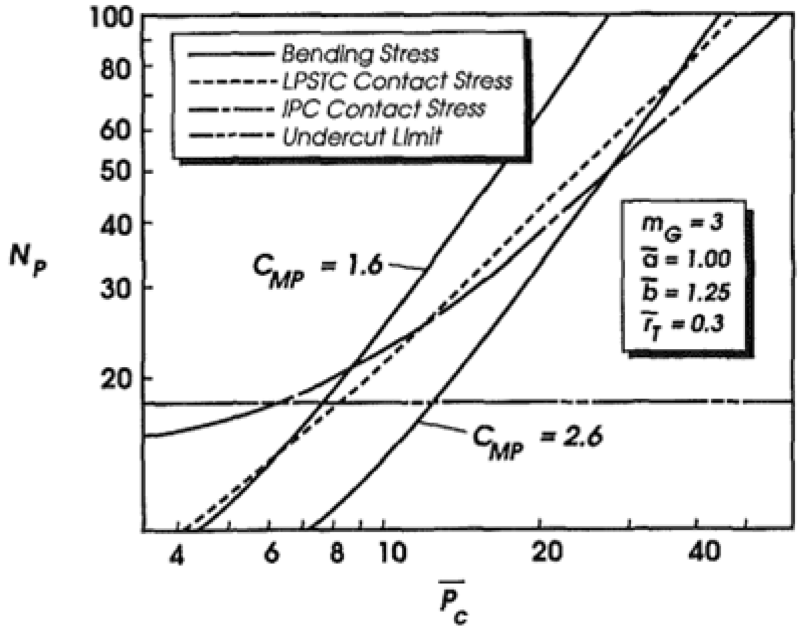


Figure 2.10. Dimensionless design space for a given set of tooth proportions (Carroll R.K. and Johnson G.E., 1989)

Table 2.13. Typical values of C_{MP} (Carroll R.K. and Johnson G.E., 1989)

Hardness	Allowable contact stress number S_{ac} (ksi)	Allowable contact stress number S_{at} (ksi)	C_{MP}^{*}
180 BHN	85 - 95	25 - 33	2,63 – 2,68
240 BHN	105 - 115	31 - 41	2,45 – 2,53
300 BHN	120 - 135	36 - 47	2,36 – 2,38
360 BHN	145 - 160	40 - 52	2,15 – 2,20
400 BHN	155 - 170	42 - 56	2,09 – 2,17
55 HRC	180 - 200	55 - 65	2,07 – 2,04
60 HRC	200 - 225	55 - 70	1,93 – 1,94
* Value based on E=30 Mpsi, v=0,3			

2.8. Summary

It has been seen that various studies are available. But these studies are generally related to verification of FEM analysis by making analytical approaches, effect of profile modifications on the gear bending stress, surface contact stress and as well as selection of module of a spur gear, and optimization related studies are available. But a comprehensive comparison of design approaches has not existed in literature yet. Because of that reason different machine element textbooks and standards for the design of spur gear have been searched. And spur gear design have been performed and compared by using five different types of design approaches with different level of difficulty.

Previous studies have also shown that comprehensive comments on the results have not been given broadly. But in this study, the design of an involute spur gear have been performed for different speed ratios and at different amount of power transmissions. Thus the effect of power transmission and speed reduction on the module and face width have been investigated by making a comparison between different types of design approaches.

3. MATERIAL AND METHOD

3.1. Material

Before starting to deal with a gear design problem, the materials of parts have to be selected. In the design of spur gear, the properties of pinion and gear materials must be in a good agreement for proper design because the mechanical properties of materials have to satisfy all service conditions.

The combination of a steel pinion and cast iron gear represent a well-balanced design. Because cast iron has low cost, ease of casting, good machinability, high wear resistance, and good noise abatement. Cast iron gears typically have greater surface fatigue strength than bending fatigue strength (Ugural A.C., 2003).

In this study, AISI 4140 oil quenched and tempered at 425 °C has been selected for pinion. And ASTM Ductile iron quenched to bainite, Grade 120-90-02 has been selected for the gear. The properties of materials for both pinion and gear have been given in Table 3.1.

Table 3.1. Material Properties of pinion and gear

Material Property	Pinion	Gear
Yield strength	1140 MPa	621 MPa
Ultimate tensile strength	1250 MPa	827 MPa
Brinell hardness number	370 HB	400 HB
Density	7850 kg/m^3	7850 kg/m^3
Poisson's ratio	0,3	0,3
Modulus of elasticity	200 GPa	170 GPa

3.2. Method

In this thesis work, design of an involute spur gear has been performed based on both bending fatigue failure and surface contact failure theories according to the five most common design approaches. These are;

1. Mechanical Engineering Design 1st Metric Edition (Shigley's J.E., 1985),
2. Shigley's Mechanical Engineering Design 9th Edition (Budynas R.G. and Nisbett J.K., 2011),
3. Fundamentals of Machine Component Design 5th Edition (Juvinall R.C. and Marshek K.M., 2011),
4. ANSI/AGMA 2101 - D04 Standard (2004) and
5. ISO Standards 6336-Part 1-3 (2006), -Part 5 (2003), -Part 6 (2004), and ISO 9085:2002 (2002).

After the calculations have been carried out for the each of the design approaches, the reliability of results have been verified by using ANSYS Workbench 14.0. Design of an involute spur gear has been achieved analytically using the most common design approaches mentioned above, then spur gears have been modeled on CATIA V5 R20 with the aid of design results (module and face width). Finally 3D models of spur gears have been subjected to gear stresses on ANSYS Workbench 14.0, and numerically obtained results have been compared with analytical calculations.

Two important design parameters, module (m) and face width (F) calculations have been carried out with the five most common design approaches mentioned above. In each of the above approaches, bending fatigue failure and surface contact failure have depended on design variables that affect the material strength and failure stresses. But different kinds of design approaches have shown that the design variables have been tackled in some different ways in each of the approach.

Module and face width are two essential parameters for sizing a gear. In this study, these two important parameters have been determined based on gear stresses called as “*bending stress*” which is occurred in tooth root, and “*surface contact stress*” which is occurred on tooth surface while a pair of gear is in a mesh. Module selection and face width determination have been performed iteratively with the aid of design variables required for determining failure stresses and material strength due to the operating conditions. When the face width reaches in a range of (3p, 5p) where p is the circular pitch (p.m), iteration is stopped and the last iteration step gives the proper module of the gear, see Figure 3.2. This procedure has been made for all types of design approaches except for the Fundamentals of Machine Component Design 5th Edition. In this textbook, the selection of module is recommended to search in a region of (9.m, 14.m), where m is the module.

In this study a comprehensive comparison has been made between five types of design approaches and to clarify this study, a flow chart has been introduced as shown in Figure 3.1.

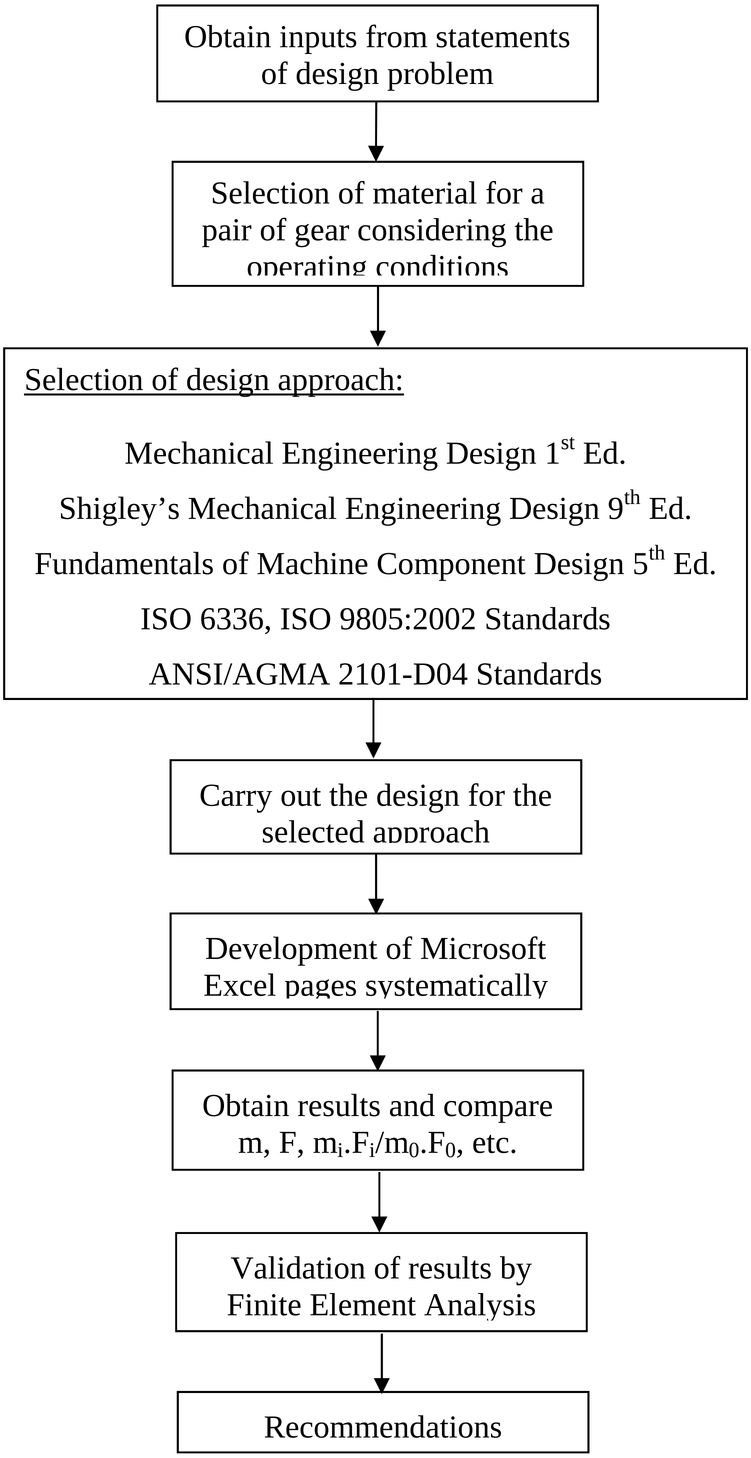


Figure 3.1. General systematic approach used for obtaining the results for the comparison of gear design approaches

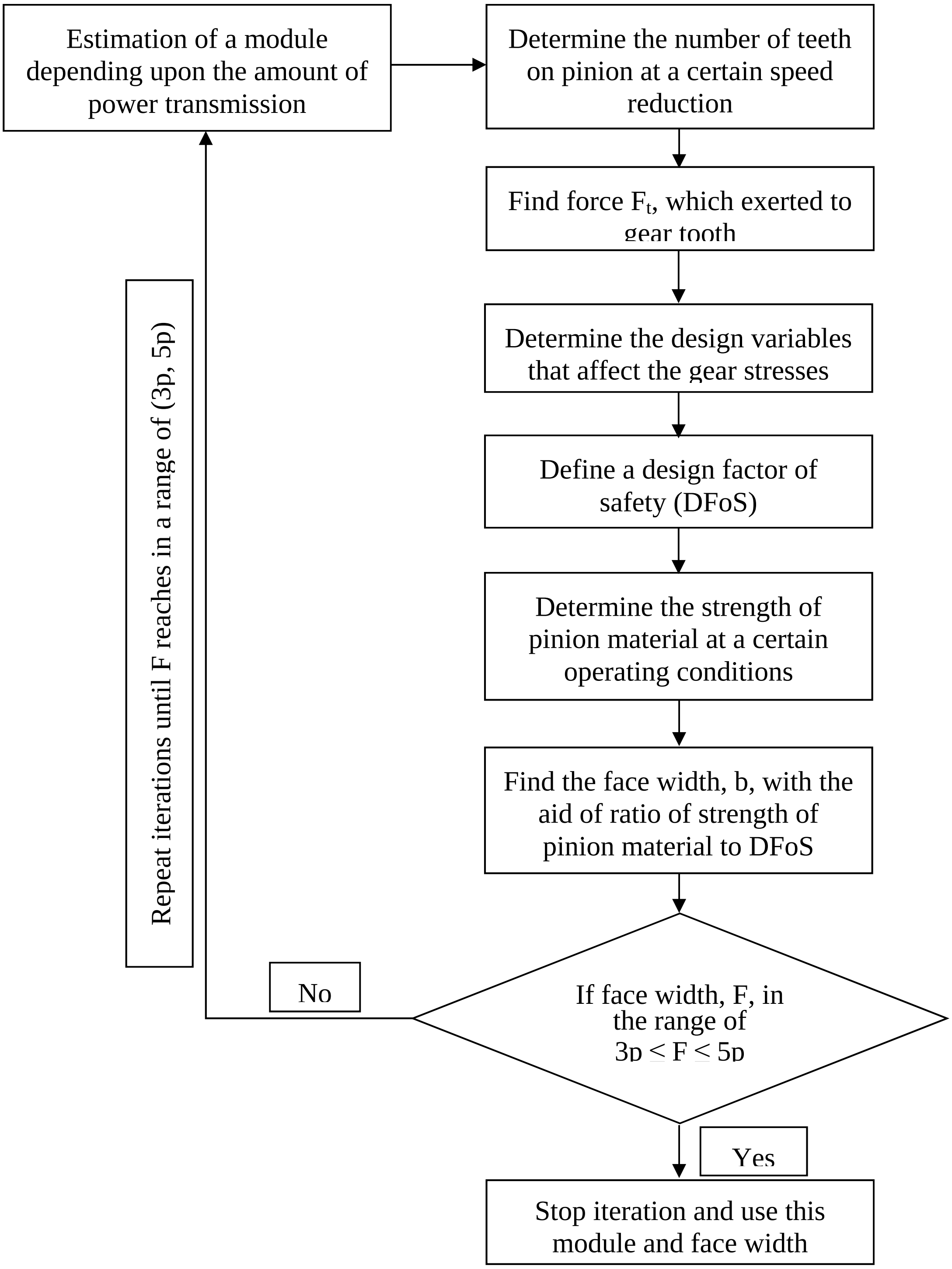


Figure 3.2. Flow chart for the design of an involute spur gear

In each type of design approach, the operating conditions such as number of cycles, gear speed ratio, gear transmission accuracy, input speed of a power source, design factor of safety, reliability, etc. have been kept identical throughout the study. This provides fair comparison of the results.

For example design factor of safety has been taken as 2,1. Design approaches given in Shigley's all Machine Elements books design factor of safety equal or greater than 2,0 is recommended (Sgihley J.E., 1985, Budynas R.G. and Nisbett J.K., 2011). In 5th Edition of Fundamentals of Machine Component Design a value of about 1,5 is recommended. In ISO 6336 - Part 3 and ISO 9085:2002 Standards suggests to select a design factor of safety by deciding between both manufacturer and user, however ISO 9085 - 2002 Standard recommends a minimum safety factor of 1,0. Also ANSI/AGMA 2101 - D04 Standard does not specify a certain value for a design factor of safety. Instead of defining a certain value for a safety, ANSI/AGMA 2101-D04 Standard recommends to use a factor by using some analysis of service experiences according to the type of industrial applications. Considering the above and providing the same conditions for the comparison of the results obtained from the each approaches a safety factor of 2,1 has been taken. These are also tabulated in Table 3.2. Finding module and face width have been made by equating gear stress equation with strength of material by considering a certain design factor of safety. Design of involute spur gear has been defined for a life cycles of 10^8.

Table 3.2. Recommended values for design factor of safety

Design Approaches	Recommended Design Factor of Safety
Mechanical Engineering Design	^{3} 2,0
Shigley's Mechanical Engineering Design	^{3} 2,0
Fundamentals of Machine Component Design	1,3 ~ 1,5
ANSI/AGMA 2101-D04 Standards	depends on service experiments according to the type of application
ISO 6336 Standards	depends on both manufacturer and user decision
ISO 9805:2002 Standards	^{3} 1,0

Gear transmission quality has been classified as machined, shaved or ground in Shigley's books (Shigley J.E., 1985, Budynas R.G. and Nisbett J.K., 2011) and Fundamentals of Machine Component Design 5th Edition (Juvinall R.C. and Marshek K.M., 2011). However in Fundamentals of Machine Component Design 5th Edition gear qualities described by symbols A to E in descending order. Symbol A meets number 9 for a gear quality level for ANSI/AGMA Standards. The gear transmission quality has been divided into 9 different classes for ANSI/AGMA 2101-D04 Standard as this is defined in 8 classes for ISO 6336-Part 3 and ISO 9085:2002 Standards. A gear transmission accuracy number of 8 and 9 also covers machined, shaved or ground conditions. In literature, the rule of 17 has been mentioned for gear quality (Chala G., 1999). 17 means the sum of the gear qualities for both ISO and ANSI/AGMA Standards. For instance, a gear quality level of 10 in ANSI/AGMA Standards is equal to a gear quality of 7 for ISO Standards. In ANSI/AGMA Standards gear quality levels increases in ascending order while in ISO Standards, increases in descending order, see Table 3.3.

Some European manufacturers employ standards of the German DIN (Deutsche Industrie Normen) system whose quality numbers are similar to those of the ISO (Mott R.L., 2003).

Table 3.3. Gear quality numbers for AGMA, ANSI/AGMA, ISO Standards (Mott R.L., 2003)

AGMA 2008	AGMA 2015	ISO 1328	AGMA 2008	AGMA 2015	ISO 1328
Q5	---	12	Q11	A6	6
Q6	A11	11	Q12	A5	5
Q7	A10	10	Q13	A4	4
Q8	A9	9	Q14	A3	3
Q9	A8	8	Q15	A2	2
Q10	A7	7	Most precise		

Since gears are used as speed reducers or to transmit power and motion all calculations have been done at a gear speed ratio of 1:1, 3:1, 5:1, 8:1 and 10:1 respectively and power transmission of 1 kW, 10 kW, 100 kW, 500 kW and 1000 kW for each of the speed ratio. All results have been plotted on a same diagram or tabulated into the same diagram for the ease of comparison. All of the calculations have been executed on Microsoft Excel pages. The results obtained from excel pages was also verified for only 1:1 gear speed ratio and at 10 kW power transmissions by using numerical finite element method, ANSYS Workbench 14.0.

In this study, only the design of pinion has been considered for the comparison of the results of the different approaches. This is because pinion is the smallest and weakest member in meshing couple and rotates more than the gear itself for the speed ratios greater than 1:1. This approach is also used very commonly for the design of gears. The work aims to determine the effect of speed ratio, therefore gear speed ratios of 1:1, 3:1, 5:1, 8:1 and 10:1 were considered and for these speed ratios the minimum number of teeth on pinion has been selected to be the same and determined at the following section considering the interference-free involute profile.

3.2.1. Determination of Interference-Free Pinion Gear Teeth Number

If the mating gear has more teeth than the pinion, then the smallest number of teeth, N_p, on the pinion without interference is given by Budynas R.G. and Nisbett J.K. (2011);

$$N_P = \frac{2.k}{(1+2.m).\sin^2\emptyset}.\left(m+\sqrt{m^2+(1+2.m).\sin^2\emptyset}\right)$$

Where $m = m_G = N_G / N_P$

The speed ratio for 1:1, the number of teeth on both pinion and gear equal to each other and the minimum number of teeth can be determined as follow;

$$N_P = \frac{2.k}{3.\sin^2\emptyset}.\left(1+\sqrt{1+3.\sin^2\emptyset}\right)$$

For a full depth teeth k = 1,0 and with the pressure angle of Æ = 20° then the N_P has been represented from following table.

Table 3.4. Minimum number of teeth on pinion for various speed ratio

Speed ratio	Minimum number of teeth on pinion
1 : 1	13
3 : 1	15
5 : 1	16
8 : 1	17
10 : 1	17

Literature research has been shown that spur gears are used as a speed reducer till 10:1 (Berg Manufacturing, Gear Reference Guide). Therefore calculations have been carried out with a range from 1:1 to 10:1 speed reduction.

Now in the following sections, design of an involute spur (pinion) gear has been described for each of the design approaches.

3.2.2. Spur Gear Design Based on Bending Fatigue Failure

3.2.2.1. Design Approach Using Mechanical Engineering Design 1st Metric Edition

In this design approach, permissible bending stress has been equalized to endurance limit of gear tooth by considering the selected design factor of safety, n_d.

$$\sigma_p = \frac{W_t}{K_v.F.m.J} \tag{3.1.}$$

Where

σ_p: Permissible bending stress, in MPa

W_t: Tangential component of load, in N

K_v: Dynamic factor

F: Face width of gear tooth, in mm

m: Normal module of gear, in mm

J: Geometry factor

Transmitted load, Wt, is calculated as;

$$W_t = \frac{H}{V} \tag{3.2.}$$

Where H is the transmitted power in Watt, and V is the pitch line velocity in m/s, calculated as;

$$V = \frac{\pi.d.n}{60} \tag{3.3.}$$

Where d is the pitch diameter in the unit of meter, and n is the input speed of power source in rev/min.

For the gears which have high precision shaved or ground teeth and if an appreciable dynamic load is developed then the dynamic factor is calculated as;

$$K_v = \left[\frac{78}{78+(200.V)^{1/2}}\right]^{1/2} \tag{3.4.}$$

The AGMA established a factor J, called geometry factor, which uses the modified form factor Y. Values of geometry factor J are given in Table 3.5.

Table 3.5. AGMA geometry factor J for teeth having Æ = 20° , a = 1m, b = 1,25m, and r_f = 0,300m (Shigley J.E., 1985)

Num-ber of teeth	**Number of teeth in mating gear**							
	1	**17**	**25**	**35**	**50**	**85**	**300**	**1000**
18	0,24486	0,32404	0,33214	0,33840	0,34404	0,35050	0,35594	0,36112
19	0,24794	0,33029	0,33878	0,34537	0,35134	0,35822	0,36405	0,36963
20	0,25072	0,33600	0,34485	0,35176	0,35804	0,36532	0,37151	0,37749
21	0,25323	0,34124	0,35044	0,35764	0,36422	0,37186	0,37841	0,38475
22	0,25552	0,34607	0,35559	0,36306	0,36992	0,37792	0,38479	0,39148
24	0,25951	0,35468	0,36477	0,37275	0,38012	0,38877	0,39626	0,40360
26	0,26289	0,36860	0,37272	0,38115	0,38897	0,39821	0,40625	0,41418
28	0,26580	0,37462	0,37967	0,38851	0,39673	0,40650	0,41504	0,42351
30	0,26831	0,38394	0,38580	0,39500	0,40359	0,41383	0,42283	0,43179
34	0,27247	0,39170	0,39671	0,40594	0,41517	0,42624	0,43604	0,44586
38	0,27575	0,40223	0,40446	0,41480	0,42456	0,43633	0,44680	0,45735
45	0,28013	0,40808	0,41579	0,42685	0,43735	0,45010	0,46152	0,47310
50	0,28252	0,41702	0,42208	0,43555	0,44448	0,45778	0,46975	0,48193
60	0,28613	0,42620	0,43173	0,44383	0,45542	0,44696	0,48243	0,49557
75	0,28979	0,43561	0,44163	0,45440	0,46668	0,48179	0,49554	0,50970
100	0,29353	0,44530	0,45180	0,46527	0,47827	0,49437	0,50909	0,52435
150	0,29738	0,44530	0,46226	0,47645	0,49023	0,50736	0,52312	0,53954
300	0,30141	0,45523	0,47304	0,48798	0,50256	0,52078	0,53765	0,55533
Rack	0,30571	0,46554	0,48415	0,49988	0,51529	0,53467	0,55272	0,57173

Endurance limits for the gear materials is considered as follow;

$$S_e = k_a . k_b . k_c . k_d . k_e . k_f . S'_e \quad (3.5.)$$

Where

S_e: Endurance limit of gear tooth,

k_a: Surface factor, Figure 3.3

k_b: Size factor, Table 3.6

k_c: Reliability factor, Table 3.7

k_d: Temperature factor, Equation 3.6

k_e: Modifying factor for stress concentration,

k_f: Miscellaneous effects factor, Figure 3.4

$S_e^{'}$: Endurance limit of rotating beam specimen.

The surface factor, k_a, should always correspond to a machined finish, even when the flank of the tooth is ground or shaved. The reason for this is that the bottom land is usually not ground, but left as the original machined finished (Shigley J.E., 1985).

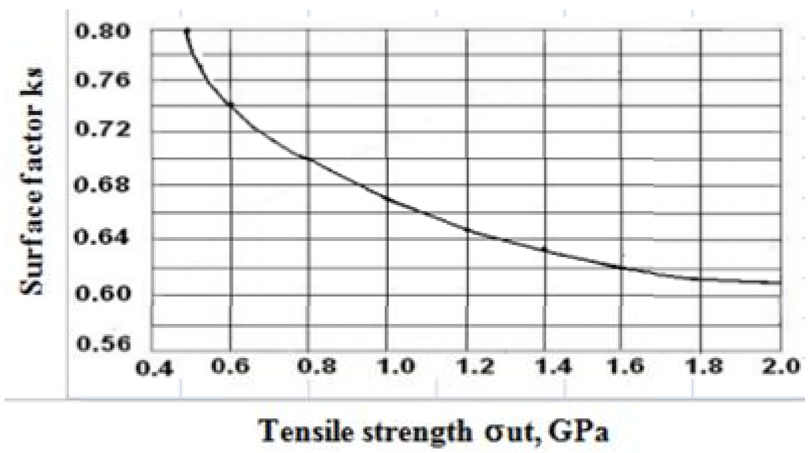


Figure 3.3. Surface finish factor k_a for cut, shaved, and ground gear teeth (Shigley J.E., 1985)

Table 3.6. Size factors for spur gear teeth (Shigley J.E., 1985)

Module m	**Factor k_b**	**Module m**	**Factor k_b**
1 to 2	1,000	11	0,843
2,25	0,984	12	0,836
2,5	0,974	14	0,824
2,75	0,965	16	0,813
3	0,956	18	0,804
3,5	0,942	20	0,796
4	0,930	22	0,788
4,5	0,920	25	0,779
5	0,910	28	0,770
5,5	0,902	32	0,760
6	0,894	36	0,752
7	0,881	40	0,744
8	0,870	45	0,736
9	0,960	50	0,728
10	0,851		

Table 3.7. Reliability factors (Shigley J.E., 1985)

Reliability R	0,50	0,90	0,95	0,99	0,999	0,9999
Reliability Factor k_c	1,000	0,897	0,868	0,814	0,753	0,702

For paying particular attention to the limitations, temperature factor equation is;

$$k_d=\begin{cases} 1 & T \leq 350 \\ 0,5 & 350 < T \leq 500 \end{cases} \tag{3.6.}$$

Where T is in degrees Celsius.

The fatigue stress concentration factor K_f has been incorporated into the geometry factor J. Since it is fully accounted for use, k_e=1,00 for gears.

Gears that always rotate in the same direction and are not idlers are subjected to a tooth force that always acts on the same side of the tooth. Thus the fatigue load is repeated but not reversed and so the tooth is said to be subjected to one way bending (Shigley J.E., 1985).

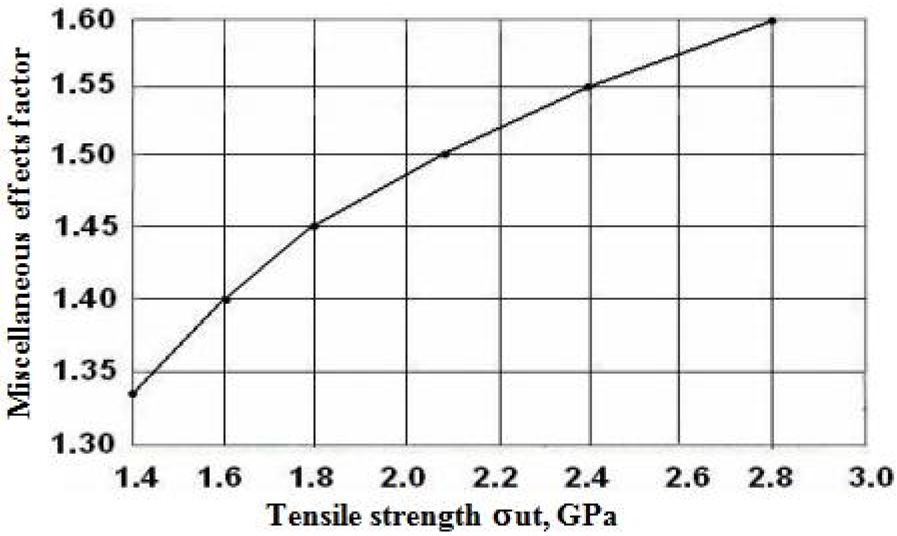


Figure 3.4. Miscellaneous effects factor k_f (Shigley J.E., 1985)

For one way bending use k_f=1,33 for values of S_{ut} less than 1,4 GPa.

For two way bending$k_f = 1{,}00$

Endurance limit of rotating beam specimen;

$$S_e^{'} = 0{,}5.S_{ut} \quad \text{when} \quad S_{ut} \leq 1400 \text{ Mpa} \tag{3.7.}$$

The formula for factor of safety;

$$n_G = K_o.K_m.n_d \tag{3.8.}$$

In this formula K_o is the overload factor, recommended values are listed in Table 3.8. Factor K_m is an AGMA load distribution factor which accounts for the possibility that the tooth force may not be uniformly distributed across the full face width.

Table 3.9 is used for K_m. The factor n_d in equation is the usual design factor of safety and it is recommended to use equal or greater than 2,0 to guard against fatigue failure.

Table 3.8. Overload correction factor K_o (Shigley J.E., 1985)

Source of Power	**Driven Machinery**		
	Uniform	**Moderate Shock**	**Heavy Shock**
Uniform	1,00	1,25	1,75
Light shock	1,25	1,50	2,00
Medium shock	1,50	1,75	2,25

Table 3.9. Load distribution factor K_m for spur gears (Shigley J.E., 1985)

Characteristics of Support	**Face Width (mm)**			
	0 - 50	**150**	**225**	**400 up**
Accurate mountings, small bearing clearances, minimum deflection, precision gears	1,3	1,4	1,5	1,8
Less rigid mountings, less accurate gears, contact across the full face	1,6	1,7	1,8	2,2
Accuracy and mounting such that less than full-face contact exists	Over 2,2	Over 2,2	Over 2,2	Over 2,2

After determining the factors, following equation is used to determine face width;

$$s_p = \frac{S_e}{n_G} \qquad (3.9.)$$

Now, in the design, a module is selected and F is found. The suitability of the F is checked by considering the common acceptance of $3p \leq F \leq 5p$ range, if F is not in between the (3p, 5p), the iteration is continued by selecting the next choice of module. Else, the iteration is ended and the module and face width is recorded as an accepted proper solution (Shigley J.E., 1985).

3.2.2.2. Design Approach Using Shigley's Mechanical Engineering Design 9th Edition

The procedure is mainly similar to the previous one but some differences exist for the factors. Failure by bending will occur when the significant tooth stress equals or exceeds either the yield strength or the bending endurance strength. Allowable bending stress has been equalized to fully corrected endurance strength of gear tooth by considering the selected design factor of safety.

$$\sigma_{all}=\frac{K_v.W^t}{F.m.Y} \qquad (3.10.)$$

And

$$S_e=n_d.\sigma_{all} \qquad (3.11.)$$

Where

s_{all}: Allowable bending stress

Wt: Tangential component of load, in N

K_v: Dynamic factor

F: Face width, in mm

m: Module, in mm

Y: Lewis form factor

S_e: Fully corrected endurance strength

n_d: Design factor of safety

When a pair of gears is driven at moderate or high speed and noise is generated, it is certain that dynamic effects are present. For gears with shaved or ground profile;

$$K_v=\sqrt{\frac{5,56+\sqrt{V}}{5,56}} \tag{3.12.}$$

Lewis form factor, Y, is determined from the Table 3.10.

Table 3.10. Values of the Lewis Form Factor Y (These Values Are for a Normal Pressure Angle of 20°, Full Depth Teeth) (Budynas R.G. and Nisbett J.K., 2011)

Number of Teeth	Y	Number of Teeth	Y
12	0,245	28	0,353
13	0,261	30	0,359
14	0,277	34	0,371
15	0,29	38	0,384
16	0,296	43	0,397
17	0,303	50	0,409
18	0,309	60	0,422
19	0,314	75	0,435
20	0,322	100	0,447
21	0,328	150	0,46
22	0,331	300	0,472
24	0,337	400	0,48
26	0,346	Rack	0,485

Fully corrected endurance strength is calculated as;

$$S_e = k_a . k_b . k_c . k_d . k_e . k_f . S_e' \tag{3.13.}$$

Where

k_a: Surface condition modification factor,

k_b: Size modification factor

k_c: Load modification factor

k_d: Temperature modification factor

k_e: Reliability factor

k_f: Miscellaneous effects modification factor

$S_e^{'}$: Rotary-beam test specimen endurance limit

Surface factor, ka;

$$k_a = a.S_{ut}^b, \tag{3.14.}$$

Where a and b are determined from Table 3.11.

Table 3.11. Parameters for marin surface modification factor (Budynas R.G. and Nisbett J.K., 2011)

Surface Finish	**Factor, a**		**Exponent, b**
	S_{ut}, kpsi	**S_{ut}, Mpa**	
Ground	1,34	1,58	-0,085
Machined or cold-drawn	2,7	4,51	-0,265
Hot-rolled	14,4	57,7	-0,718
As-forged	39,9	272	-0,995

Size factor, k_b;

$$k_b = 0,904.(b.m.\sqrt{Y})^{0,035} \tag{3.15.}$$

Where b is the face width, m is the module and Y is the Lewis form factor.

Loading factor, k_c = 1 for bending.

Temperature and reliability factors, $k_d = k_e = 1$ are selected to be unity as the room temperature and %50 of reliability are considered throughout the design comparisons.

Miscellaneous effects factor for stress concentration;

$k_f = 1{,}66.k_f^{'}$ (3.16.)

$k_f = \frac{1}{K_f}$ (3.17.)

$K_f = 1 + q.(K_t - 1)$ (3.18.)

From Figure 3.5 with

$$\frac{r}{d} = \frac{r_f}{t}$$

Where “m”, is the module and “t” is the thickness of tooth equals to half of circular pitch (here m Since D/d = ∞, it is approximated as D/d = 3.

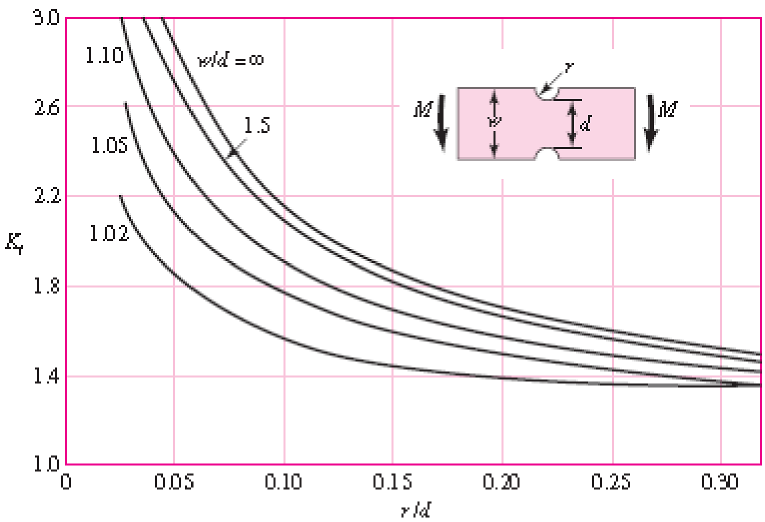


Figure 3.5. Stress concentration factor, K_t

After determining the K_t notch sensitivity, q, is read from Figure 3.6 for a 20°full depth tooth the radius of the root fillet is denoted r_f= 0,3.m

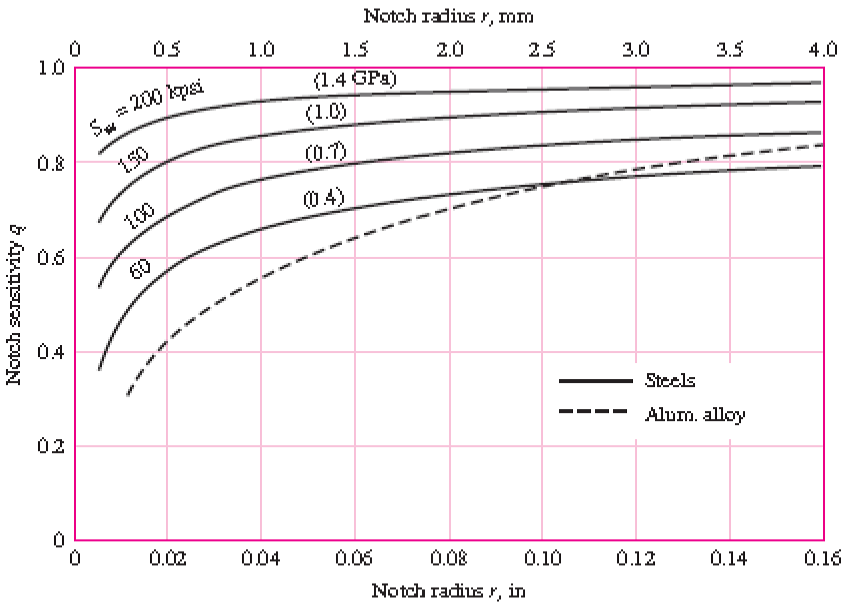


Figure 3.6. Notch-sensitivity charts. For larger notch radii, use the values of q corresponding to the r = 4 mm

Rotary-beam test specimen endurance limit is determined as follow;

$$S'_e = 0{,}5.S_{ut} \text{ for } S_{ut} < 1400 \text{ MPa} \quad (3.19.)$$

The textbook recommends that the Equation 3.10 is important because it forms the basis for the AGMA approach to the bending strength of gear teeth. It is in general use for estimating the capacity of gear drives when life and reliability are not important considerations. The equations can be useful in obtaining a preliminary estimate of gear sizes needed for various applications (Budynas R.G. and Nisbett J.K., 2011).

3.2.2.3. Design Approach Using Fundamentals of Machine Component Design 5th Edition

The design approach given by Juvinall and Marshek slightly differs to the previous ones for bending fatigue failure. This approach recommends that in the absence of more specific information, the factors affecting gear tooth bending stress can be taken into account by embellishing the Lewis equation to the following form;

$$\sigma = \frac{F_t}{m.b.J}.K_v.K_o.K_m \qquad (3.20.)$$

Where

s: Bending fatigue stress,

m: Module,

b: Face width,

J: Spur gear geometry factor, determined from Figure 3.7. This factor includes the Lewis form factor Y and also a stress concentration factor.

K_v: Velocity or dynamic factor that indicating the severity of impact as successive pairs of teeth engage. This is a function of pitch line velocity and manufacturing accuracy. Gears with shaved or ground profile, it is calculated from Equation 3.12.

K_o: Overload factor that reflecting the degree of non-uniformity of driving and load torques. In the absence of better information, the values in Table 3.8 have long been used as a basis for rough estimates.

K_m: Mounting factor that reflecting the accuracy of mating gear alignment. Table 3.12 is used as a basis for rough estimates.

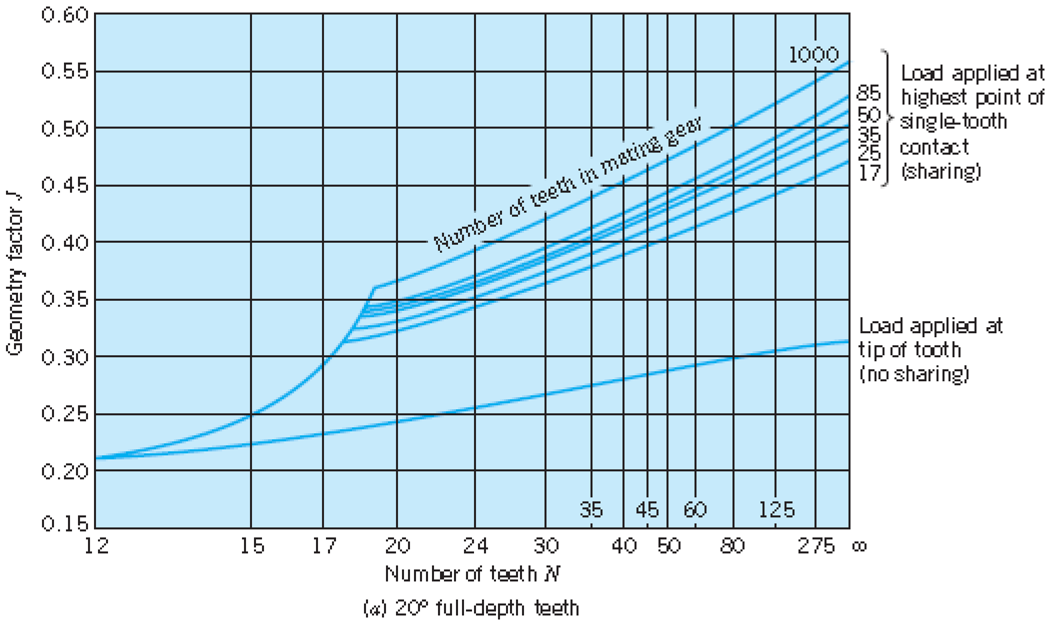


Figure 3.7. Geometry factor J for standard spur gears (Juvinall R.C., Marshek K.M., 2011)

Table 3.12. Mounting Correction Factor K_m (Juvinall R.C., Marshek K.M., 2011)

Characteristics of Support	Face Width (in.)			
	0 to 2	**6**	**9**	**16 up**
Accurate mountings, small bearing clearances, minimum deflection, precision gears	1,3	1,4	1,5	1,8
Less rigid mountings, less accurate gears, contact across the full face	1,6	1,7	1,8	2,2
Accuracy and mounting such that less than full-face contact exists	over 2,2			

The effective fatigue stress from Equation 3.20 must be compared with the corresponding fatigue strength. For infinite life, the appropriate endurance limit is estimated from the following equation;

$$S_n = S'_n . C_L . C_G . C_S . k_r . k_t . k_{ms} \tag{3.21.}$$

Where

S'_n: Standard R. R. Moore endurance limit.

For steel $S'_n = (0,5).S_{ut}$ and

for other ductile materials $S'_n = (0,7).S_{ut}$

C_L: Load factor = 1,0 for bending loads

C_G: Gradient factor = 1,0 for P>5 (m<0,2), and 0,85 for P≤5 (m≥0,2)

C_S: Surface factor, Figure 3.8. Be sure that this pertains to the surface in the fillet, where a fatigue crack would likely start. (In the absence of specific information, assume this to be equivalent to a machined surface).

k_r: Reliability factor from Table 3.13.

k_t: Temperature factor. For steel gears use k_t = 1,0 if the temperature (usually estimated on the basis of lubricant temperature) is less than 160°F. If not, and in the absence of better information, use

$$k_t = \frac{620}{460 + T} \quad \text{for } T > 160\ °F$$

k_{ms}: Mean stress factor. Use 1,0 for idler gears (subjected to two way bending) and 1,4 for input and output gears (one way bending).

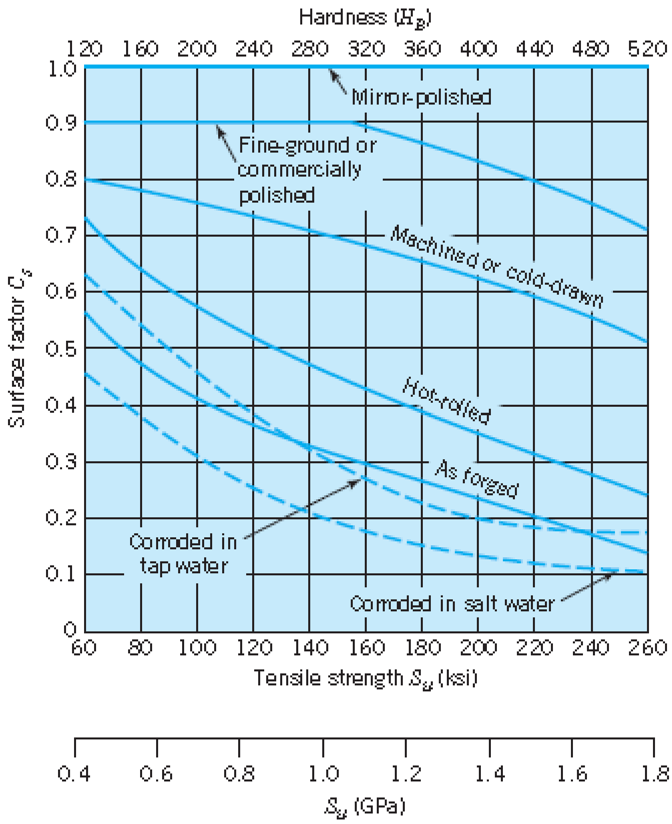


Figure 3.8. Surface factor C_S (Juvinall R.C., Marshek K.M., 2011)

Table 3.13. Reliability factor, k_r (Juvinall R.C., Marshek K.M., 2011)

Reliability (%)	50	90	99	99,9	99,99	99,999
Factor k_r	1,000	0,897	0,814	0,753	0,702	0,659

This approach recommends that the design factor of safety for bending fatigue can be taken as the ratio of fatigue strength to fatigue stress. Since factors K_o, K_m, and k_r have been taken into account separately, the design factor of safety need not be as large as would otherwise be necessary. Typically, a safety factor of 1,5 might be selected, together with a reliability factor corresponding to 99,9 percent reliability (Juvinall R.C., Marshek K.M., 2011). But in this study it is aimed to use a design factor as 2,1 for all the design approaches in order to compare the approaches at the same conditions.

3.2.2.4. Design Approach Using ISO Standards 6336 - Part 3

ISO provides gear design standards with standard number of 6336. ISO 6336 Standards has been released in 1996 which modified from DIN 3990. Since the ISO standards evolved from DIN 3990, there is a strong similarity between two (Beckman K.O., Patel V.P., 2000).

In this standard, the maximum tensile stress at the tooth root may not exceed the permissible bending stress for the material. This is the basis for rating the bending strength of gear teeth. The actual tooth root stress s_F and the permissible tooth root bending stress s_{FP} shall be calculated separately for pinion and wheel; s_F shall be less than s_{FP} (ISO 6336-Part 3, 2006).

ISO Standard 6336 - Part 3 is related to calculation of tooth bending strength, but some modifying factors to determine the bending stress are included in ISO Standards 6336 - Part 1, -Part 5, and -Part 6.

This ISO Standards give three methods to calculate these factors included in parts. These methods are mentioned as A, B or C in decreasing order of accuracy. Method A often includes full size testing as would be appropriate in the aerospace industry. Method B uses detailed calculations to correlate field data to similar designs and is the method typically used in the industrial gear market. Method C is a simplified method used for narrow applications (Beckman K.O., Patel V.P., 2000).

Standard DIN 3990, which was the base for ISO 6336, proposes five methods: A, B, C, D and E. Methods A, B and C of both the ISO and DIN standards are most frequently used (Jelaska D.T, 2012).

Tooth root stress s_F is the maximum tensile stress at the surface in the root.

Tooth root stress is calculated as

$$\sigma_F=\sigma_{F0}.K_A.K_V.K_{Fb}.K_{Fa}\leq\sigma_{FP} \qquad (3.22.)$$

With

$$\sigma_{F0}=\frac{F_t}{b.m_n}.Y_F.Y_S.Y_b.Y_B.Y_{DT} \qquad (3.23.)$$

Where

s_{F0}: Nominal tooth root stress, which is the maximum local principal stress produced at the tooth root

s_{FP}: Permissible bending stress

K_A: Application factor

K_V: Dynamic factor

K_{Fb}: Face load factor for tooth root stress

K_{Fa}: Transverse load factor for tooth root stress

F_t: Nominal tangential load

b: Face width

m_n: Normal module

Y_F: Form factor

Y_S: Stress correction factor

Y_b: Helix angle factor

Y_B: Rim thickness factor

Y_{DT}: Deep tooth factor

The application factor, K_A, adjusts the nominal load F_t in order to compensate for incremental gear loads from external sources. These additional loads are largely dependent on the characteristics of the driving and driven machines, as well as the masses and stiffness of the system, including shafts and couplings used in service. The value of K_A is determined from Table 3.14 which is obtained from ISO Standard 6336 - Part 6.

Table 3.14. Application factor, K_A (ISO 6336 Part 6, 2004)

Working characteristics of the driving machine	**Working characteristics of the driven machine**			
	Uniform	**Light shocks**	**Moderate shocks**	**Heavy shocks**
Uniform	1	1,25	1,5	1,75
Light shocks	1,1	1,35	1,6	1,85
Moderate shocks	1,25	1,5	1,75	2
Heavy shocks	1,5	1,75	2	2,25 or higher

The internal dynamic factor, K_V, relates the total tooth load, including internal dynamic effects of a multi resonance system, to the transmitted tangential tooth load. The internal dynamic factor makes allowance for the effects of gear tooth accuracy grade as related to speed and load (ISO 6336-Part 3, 2006).

$$K_v=1+\left[\frac{K_1}{K_A.\frac{F_t}{b}}+K_2\right].\frac{v.z_1}{100}.K_3.\sqrt{\frac{u^2}{1+u^2}} \quad (3.24.)$$

Table 3.15. Values of factors K_1 and K_2 for calculation of K_v (ISO 6336 Part 1, 2006)

	K_1, Accuracy grades as specified in ISO 1328-1										**K_2,** All accuracy grades
	3	4	5	6	7	8	9	10	11	12	
Spur gears	2,1	3,9	7,5	14,9	26,8	39,1	52,8	76,6	102,6	146,3	0,0193
Helical gears	1,9	3,5	6,7	13,3	23,9	34,8	47,0	68,2	91,4	130,3	0,0087

To find K_3;

$$\text{if } \frac{v.z_1}{100}.\sqrt{\frac{u^2}{1+u^2}} \leq 0{,}2 \quad \rightarrow \quad K_3=2$$

$$\text{if } \frac{v.z_1}{100}.\sqrt{\frac{u^2}{1+u^2}} \geq 0{,}2 \quad \rightarrow \quad K_3=-0{,}357.\frac{v.z_1}{100}.\sqrt{\frac{u^2}{1+u^2}}+2{,}071 \tag{3.25.}$$

The face load factors, $K_{F\beta}$ and $K_{H\beta}$, takes into account the effects of the non-uniform distribution of load over the gear face width on the surface stress ($K_{H\beta}$) and on the tooth root stress ($K_{F\beta}$).

ISO Standard 9085:2002 suggests for gear pairs without helix correction and crowning, the minimum value for $K_{H\beta}$ is 1,25 for lowest speed stages (also for single reduction gear drives) and 1,45 for all other stages. For the calculation of $K_{F\beta}$;

$$K_{F\beta}=\left(K_{H\beta}\right)^{N_F} \tag{3.26.}$$

$$N_F=\frac{(b/h)^2}{1+b/h+(b/h)^2}=\frac{1}{1+h/b+(h/b)^2} \tag{3.27.}$$

The transverse load factors, $K_{F\alpha}$ for surface stress and $K_{H\alpha}$ for tooth root stress, account for the effect of the non-uniform distribution of transverse load between several pairs of simultaneously contacting gear teeth as follows. The values for $K_{F\alpha}$ and $K_{H\alpha}$ are determined from Appendix A.

Form factor, Y_F, which takes into account the influence on nominal tooth root stress of the tooth form with load applied at the outer point of single pair tooth contact.

The stress correction factor, Y_S, is used to convert the nominal tooth root stress to local tooth root stress and, by means of this factor, the stress amplifying effect of section change at the fillet radius at the tooth root is taken into consideration. And this factor evaluates the true stress at the tooth root, critical section is more complex than the simple system evaluation presented, with evidence indicating that the intensity of the local stress at the tooth root consists of two components, one of which is directly influenced by the value of the bending moment and the other increasing with closer proximity to the critical section of the determinant position of load application (ISO 6336-Part 3, 2006)

Form factor, Y_F, the stress correction factor, Y_S, are determined considering the number of teeth and profile shifting factor from Appendix B and Appendix C respectively.

For spur gears the helix factor, Y_β, equals to 1,0.

The rim thickness factor, Y_B, is a simplified factor used to de-rate thin rimmed gears when detailed calculations of stresses in both tension and compression or experience are not available. For critically loaded applications this method should be replaced by a more comprehensive analysis. Y_B can be calculated using the following equations;

$$\text{If } \frac{s_R}{h_t} \geq 1{,}2 \quad \text{then} Y_B = 1{,}0$$

$$\text{If } \frac{s_R}{h_t} > 0{,}5 \text{ and } \frac{s_R}{h_t} < 1{,}2 \quad \text{then} Y_B = 1{,}6.\ln\left(2{,}242.\frac{h_t}{s_R}\right)$$

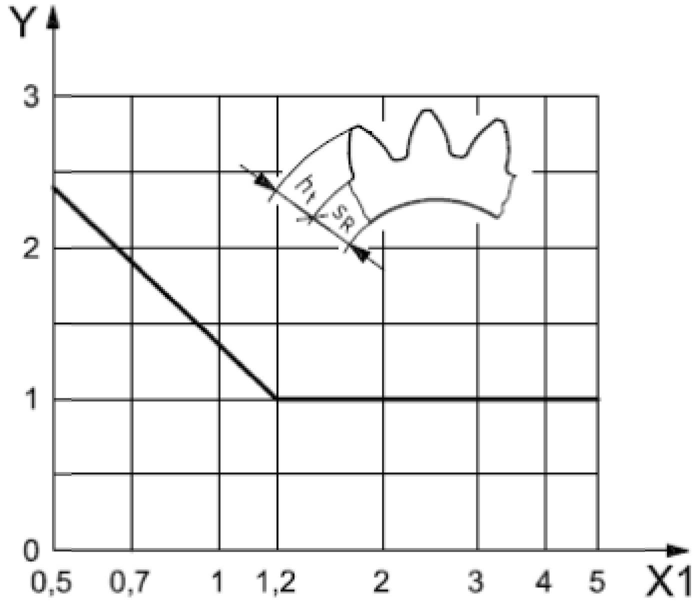


Figure 3.9.Value of rim thickness factor (ISO 6336 Part 3, 2006)

X1 backup ratio, s_R / h_t

Y rim thickness factor, Y_B

For gears of high precision (accuracy grade ≤ 4) with contact ratios in the range of $2 \leq e_{an} < 2,5$ and with applied actual profile modification to obtain a trapezoidal load distribution along the path of contact, the nominal tooth root stress s_{F0} is adjusted by the deep tooth factor, Y_{DT}.

Figure 3.10. Value of deep tooth factor (ISO 6336 Part 3, 2006)

X	virtual contact ratio, e_{an}
Y	deep tooth factor, Y_{DT}
a	Accuracy grade > 4
b	Accuracy grade ≤ 4

Permissible tooth root bending stress, σ_{FP}, is calculated as;

$$\sigma_{FP}=\frac{\sigma_{F_{lim}}.Y_{ST}.Y_{NT}}{S_{F_{min}}}.Y_{\delta relT}.Y_{RrelT}.Y_X \qquad (3.28.)$$

Where

$\sigma_{F\,lim}$: Nominal stress number (bending) from reference test gears
Y_{ST}: Stress correction factor
Y_{NT}: Life factor for tooth root stress
$S_{F\,min}$: Minimum required safety factor for tooth root stress
$Y_{\delta rel\,T}$: Relative notch sensitivity factor
$Y_{R\,rel\,T}$: Relative surface factor
Y_X: Size factor relevant to tooth root strength

The nominal stress number (bending), $\sigma_{F\,lim}$, was determined by testing reference test gears. It is the bending stress limit value relevant to the influences of the material, the heat treatment and the surface roughness of the test gear root fillets. ISO 6336-Part 5 provides information on commonly used gear materials, methods of heat treatment and the influence of gear quality on values for nominal stress numbers which is used for nominal stress.

$$\sigma_{F\,lim}=A.x+B \qquad (3.29.)$$

Table 3.16. Factors that affect nominal stress number (ISO 6336 Part 5, 2003)

No.	Material	Stress	Type	Abbreviation	Quality	A	B	Hardness	Min. hardness	Max. hardness
1	Normalized low carbon steels/cast steels[a]	contact	wrought normalized low carbon steels	St	ML/MQ	1,000	190	HBW	110	210
2					ME	1,520	250	HBW	110	210
3			cast steel	St (cast)	ML/MQ	0,986	131	HBW	140	210
4					ME	1,143	237	HBW	140	210
5		bending	wrought normalized low carbon steels	St	ML/MQ	0,455	69	HBW	110	210
6					ME	0,386	147	HBW	110	210
7			cast steel	St (cast)	ML/MQ	0,313	62	HBW	140	210
8					ME	0,254	137	HBW	140	210

Table 3.16. Continued

No.	Material	Stress	Type	Abbreviation	Quality	A	B	Hardness	Min. hardness	Max. hardness
9	cast iron materials	contact	black malleable cast iron	GTS (perl.)	ML/MQ ME	1,371	143	HBW	135	250
10						1,333	267	HBW	175	250
11			nodular cast iron	GGG	ML/MQ ME	1,434	211	HBW	175	300
12						1,500	250	HBW	200	300
13			gray cast iron	GG	ML/MQ ME	1,033	132	HBW	150	240
14						1,465	122	HBW	175	275
15		bending	black malleable cast iron	GTS (perl.)	ML/MQ ME	0,345	77	HBW	135	250
16						0,403	128	HBW	175	250
17			nodular cast iron	GGG	ML/MQ ME	0,350	119	HBW	175	300
18						0,380	134	HBW	200	300
19			gray cast iron	GG	ML/MQ ME	0,256	8	HBW	150	240
20						0,200	53	HBW	175	275

Table 3.16. Continued

No.	Material	Stress	Type	Abbreviation	Quality	A	B	Hardness	Min. hardness	Max. hardness
21	Through hardened wrought steels[b]	contact	carbon steels	V	ML	0,963	283	HV	135	210
22					MQ	0,925	360	HV	135	210
23					ME	0,838	432	HV	135	210
24			alloy steels	V	ML	1,313	188	HV	200	360
25					MQ	1,313	373	HV	200	360
26					ME	2,213	260	HV	200	390
27		bending	carbon steels	V	ML	0,250	108	HV	115	215
28					MQ	0,240	163	HV	115	215
29					ME	0,283	202	HV	115	215
30			alloy steels	V	ML	0,423	104	HV	200	360
31					MQ	0,425	187	HV	200	360
32					ME	0,358	231	HV	200	390

For the calculation of stress correction factor, Y_{ST}, the ISO Standard recommends that the tooth root stress limit values for materials, according to ISO 6336 - Part 5, were derived from results of tests of standard reference test gears for which either Y_{ST} = 2,0 or for which test results were recalculated to this value.

The life factor, Y_{NT}, accounts for the higher tooth root stress, which may be tolerable for a limited life (number of load cycles), as compared with the allowable stress at $3x10^6$ cycles. The number of load cycles, N_L, is defined as the number of mesh contacts, under load, of the gear tooth being analyzed. The allowable stress numbers are established for $3x10^6$ tooth load cycles at 99 % reliability.

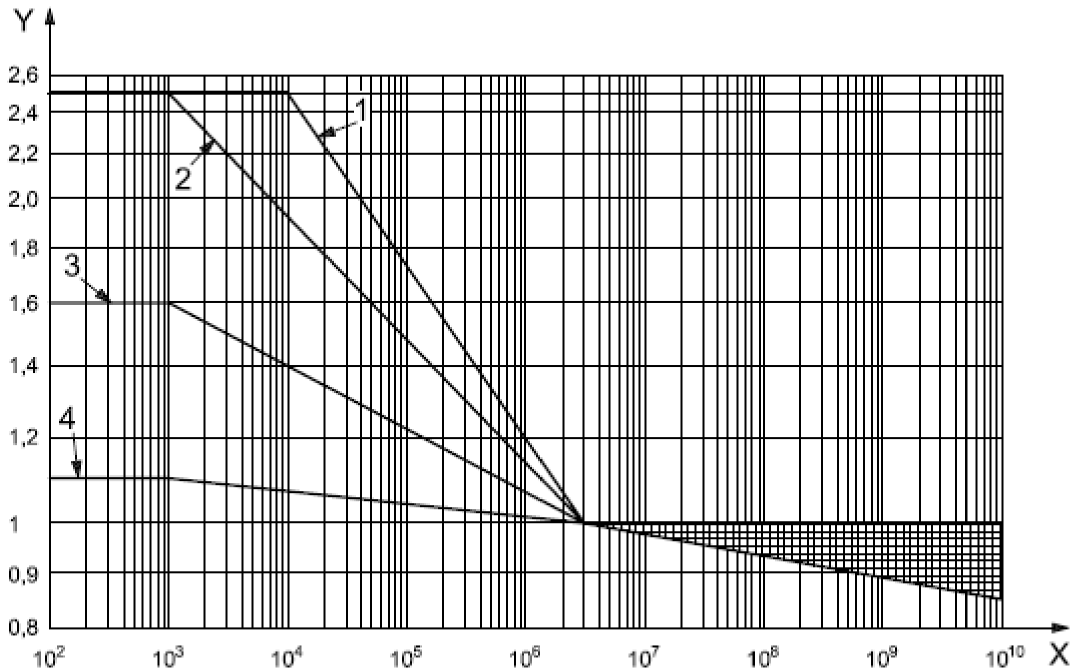


Figure 3.11. Life factor for number of load cycles (ISO 6336 Part 3, 2006)

X	number of load cycles, N_L		
Y	life factor, Y_{NT}		
1	GTS (perl.), St, V, GGG (perl. bai.)	2	Eh, IF (root)
3	NT, NV (nitr.), GGG (ferr.), GG	4	NV (nitrocar.)

Relative notch sensitivity factor, $Y_{drel\ T}$, which is the quotient of the notch sensitivity factor of the gear of interest divided by the standard test gear factor and which enables the influence of the notch sensitivity of the material to be taken into account. The reference value $Y_{drel\ T}$ = 1,0 for the standard reference test gear coincides with the stress correction factor Y_S = 2,0.

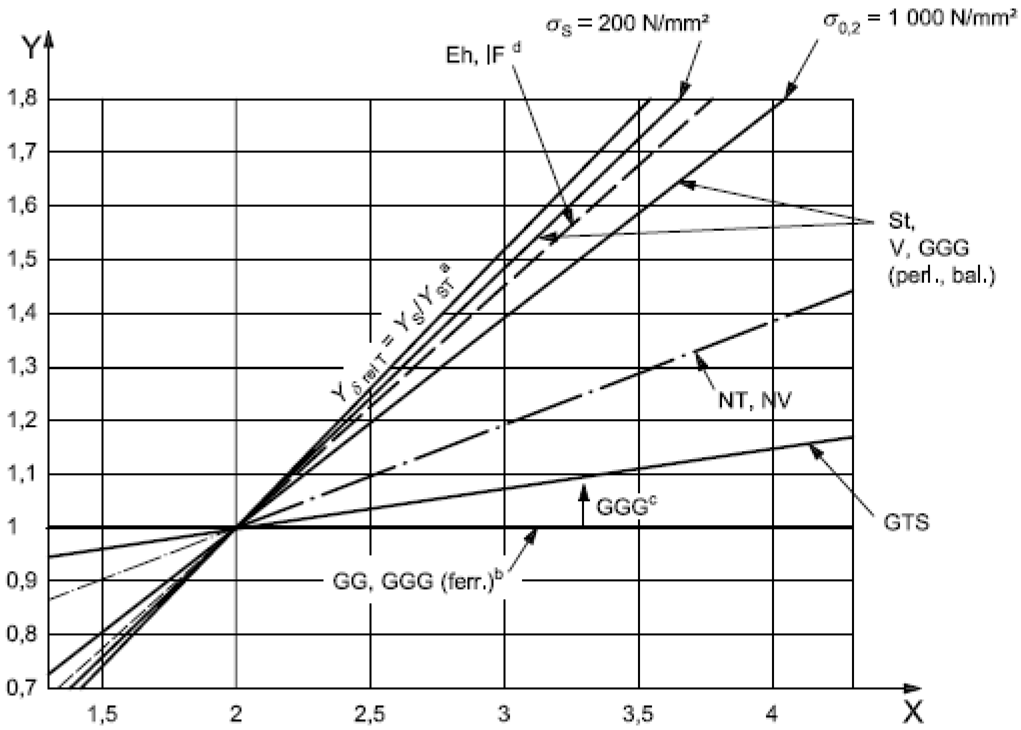


Figure 3.12. Relative notch sensitivity factor (ISO 6336 Part 3, 2006)

X stress correction factor, Y_S
Y relative notch sensitivity factor, $Y_{drel\ T}$

The surface factor, $Y_{R\ rel\ T}$, accounts for the influence on tooth root stress of the surface condition in the tooth roots. This is dependent on the material and the surface roughness in the tooth root fillets.

For V, GGG (perl., bai.), Eh and IF (root) : (3.30.)

$Y_{R\ rel\ T} = 1{,}12$

For St :

$Y_{R\ rel\ T} = 1{,}07$

For GG, GGG (ferr.) and NT, NV :

$Y_{R\ rel\ T} = 1{,}025$

The size factor, Y_X, is used to take into consideration on the influence of size on the probable distribution of weak points in the structure of the material, the stress gradients, which, in accordance with strength of materials theory, decrease with increasing dimensions, the quality of the material as determined by the extent and effectiveness of forging, the presence of defects, etc.

Table 3.17. Size factor (root), Y_X (ISO 6336 Part 3, 2006)

Material[a]		**Normal module, m_n**	**Size factor, Y_X**
St, V, GGG (perl.,bai.), GTS (perl.)	For 3×10^6 cycles	$m_n \leq 5$	$Y_X = 1,0$
		$5 < m_n < 30$	$Y_X = 1,03 - 0,006.m_n$
		$30 \leq m_n$	$Y_X = 0,85$
Eh, IF (root), NT, NV		$m_n \leq 5$	$Y_X = 1,0$
		$5 < m_n < 25$	$Y_X = 1,05 - 0,01.m_n$
		$25 \leq m_n$	$Y_X = 0,8$
GG, GGG (ferr.)		$m_n \leq 5$	$Y_X = 1,0$
		$5 < m_n < 25$	$Y_X = 1,075 - 0,015.m_n$
		$25 \leq m_n$	$Y_X = 0,7$
All materials for static stress		---	$Y_X = 1,0$

[a] See Appendix D for an explanation of the abbreviations used (ISO 6336-1:2006).

3.2.2.5. Design Approach Using ANSI/AGMA 2101 - D04 Standards

ANSI/AGMA 2101 - D04 Standards provide a simpler gear design approach than ISO 6336. The standard recommends that bending stress or bending stress number have to be equal or less than the allowable bending stress number. The fundamental formula for bending stress number in a gear tooth is given by ANSI/AGMA 2101 - D04:

$$\sigma_F = F_t . K_o . K_V . K_S . \frac{1}{b . m_t} . \frac{K_H . K_B}{Y_J} \quad (3.31.)$$

Where

s_F: Bending stress number, N/mm2

Ft: Transmitted tangential load, N

K_O: Overload factor

K_V: Dynamic factor

K_S: Size factor

b: Net face width of narrowest member, mm

m_t: Transverse metric module = m_n for spur gears

K_B: Rim thickness factor

Y_J: Geometry factor for bending strength

K_H: Load distribution factor

The overload factor, K_o, is intended to make allowance for all externally applied loads in excess of the nominal tangential load F_t in a particular application. Overload factors can only be established after considerable field experience is gained in a particular application (ANSI/AGMA 2101 - D04, 2004).

In determining the overload factor, consideration should be given to the fact that many prime movers and driven equipment, individually or in combination, develop momentary peak torques appreciably greater than those determined by the nominal ratings of either the prime mover or the driven equipment. There are many possible sources of overload factors, which should be considered. Some of these are: system vibrations, acceleration torques, over speeds, variations in system operation, split path load sharing among multiple prime movers, and changes in process load conditions (ANSI/AGMA 2101 - D04, 2004). The value of K_o can be read from Table 3.8.

Dynamic factor, K_v, accounts for internally generated gear tooth loads which are induced by non-conjugate meshing action of the gear teeth. Even if the input torque and speed are constant, significant vibration of the gear masses, and therefore dynamic tooth forces, can exist (ANSI/AGMA 2101 - D04, 2004).

$$K_V=\left(\frac{A+\sqrt{196{,}85.v_t}}{A}\right)^B \tag{3.32.}$$

$A=50+56.(1{,}0-B)$ for $5 \leq Q_V \leq 11$

$B=0{,}25.(12-Q_v)^{0{,}667}$

Where

Q_v is the transmission accuracy level number

The size factor, K_S, reflects non-uniformity of material properties. It depends primarily on tooth size, diameter of parts, ratio of tooth size to diameter of part, face width, area of stress pattern, and ratio of case depth to tooth size, hardenability and heat treatment of materials. The size factor may be taken as unity for most gears, provided a proper choice of steel is made for the size of the part, and its heat treatment and hardening process (ANSI/AGMA 2101 - D04, 2004).

The load distribution factor is defined as the peak load intensity divided by the average, or uniformly distributed, load intensity; i.e., the ratio of peak to mean loading. The load distribution factor modifies the rating equations to reflect the non-uniform distribution of the load along the lines of contact. The amount of non-uniformity of the load distribution is caused by, and dependent upon manufacturing variation of gears, assembly variations of installed gears, deflections due to applied loads, distortions due to thermal and centrifugal effects (ANSI/AGMA 2101 - D04, 2004).

$K_H = f(K_{H\beta}, K_{H\alpha})$

$K_{H\alpha}$: Face load distribution factor

K_{Ha}: Transverse load distribution factor

The transverse load distribution factor accounts for the non-uniform distribution of load among the gear teeth which share the load. It is affected primarily by the correctness of the profiles of mating teeth: i.e., profile modification or profile error or both. Evaluation of the numeric value of the transverse load distribution factor is beyond the scope of this standard and it can be assumed to be unity. Therefore equation can be modified to;

$K_H = K_{H\beta}$

$$K_{H\beta} = 1 + K_{Hmc}.(K_{Hpf}.K_{Hpm} + K_{Hma}.K_{He}) \quad (3.33.)$$

Where

K_{Hmc}: Lead correction factor

K_{Hpf}: Pinion proportion factor

K_{Hpm}: Pinion proportion modifier

K_{Hma}: Mesh alignment factor

K_{He}: Mesh alignment correction factor

The lead correction factor, K_{Hmc}, modifies peak load intensity when crowning or lead modification is applied.

K_{Hmc}= 1,0 for gear with unmodified leads;

K_{Hmc}= 0,8 for gear with leads properly modified by crowning or lead correction.

The pinion proportion factor, K_{Hpf}, accounts for deflections due to load.

$$K_{Hpf}=\frac{b}{10.d_{W1}}-0{,}025 \quad \text{when } b\leq 25 \tag{3.34.}$$

$$K_{Hpf}=\frac{b}{10.d_{W1}}-0{,}0375+0{,}000492.b \quad \text{when } 25mm<b\leq 432 \tag{3.35.}$$

The pinion proportion modifier, K_{Hpm}, alters K_{Hpf}, based on the location of the pinion relative to its bearing centerline.

K_{Hpm}= 1,0 for straddle mounted pinions with $(S_1/S) < 0{,}175$

K_{Hpm}= 1,1 for straddle mounted pinions with $(S_1/S) \geq 0{,}175$

Where

S_1 = offset of the pinion; i.e., the distance from the bearing span centerline to the pinion mid face

S = bearing span; i.e., the distance between the bearing center lines

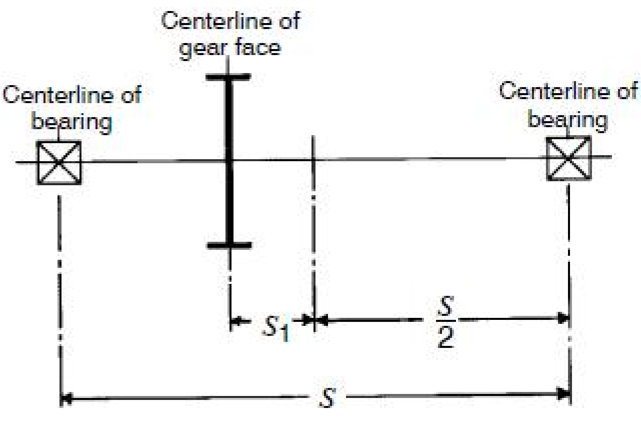


Figure 3.13. Measure of S and S1 values (ANSI/AGMA 2101 – D04, 2004)

The mesh alignment factor, K_{Hma}, accounts for the misalignment of the axes of rotation of the pitch cylinders of the mating gear elements from all causes other than elastic deformations.

$$K_{Hma}=A+B.b+C.b^2 \quad (3.36.)$$

Table 3.18. Empirical constants; A, B and C (ANSI/AGMA 2101 – D04, 2004)

Curve	A	B	C
Curve 1 Open gearing	$2{,}47\times 10^{-1}$	$0{,}657\times 10^{-3}$	$-1{,}186\times 10^{-7}$
Curve 2 Commercial enclosed gear units	$1{,}27\times 10^{-1}$	$0{,}622\times 10^{-3}$	$-1{,}69\times 10^{-7}$
Curve 3 Precision enclosed gear units	$0{,}675\times 10^{-1}$	$0{,}504\times 10^{-3}$	$-1{,}44\times 10^{-7}$
Curve 4 Extra precision enclosed gear units	0380×10^{-1}	$0{,}402\times 10^{-3}$	$-1{,}27\times 10^{-7}$

The mesh alignment correction factor, K_{He}, is used to modify the mesh alignment factor when the manufacturing or assembly techniques improve the effective mesh alignment.

K_{He} = 0,80 when the gearing is adjusted at assembly

= 0,80 when the compatibility of the gearing is improved by lapping

= 1,0 for all other conditions

When gears are lapped and mountings are adjusted at assembly, the suggested value of K_{He} is 0,80.

The rim thickness factor, K_B, adjusts the calculated bending stress number for thin rimmed gears. Where the rim thickness is not sufficient to provide full support for the tooth root, the location of bending fatigue failure may be through the gear rim, rather than at the root fillet (ANSI/AGMA 2101 - D04, 2004). It is a function of the backup ratio, m_B,

$$m_B = \frac{t_R}{h_t}$$

t_R: gear rim thickness below the tooth root, mm

h_t: gear tooth whole depth, mm

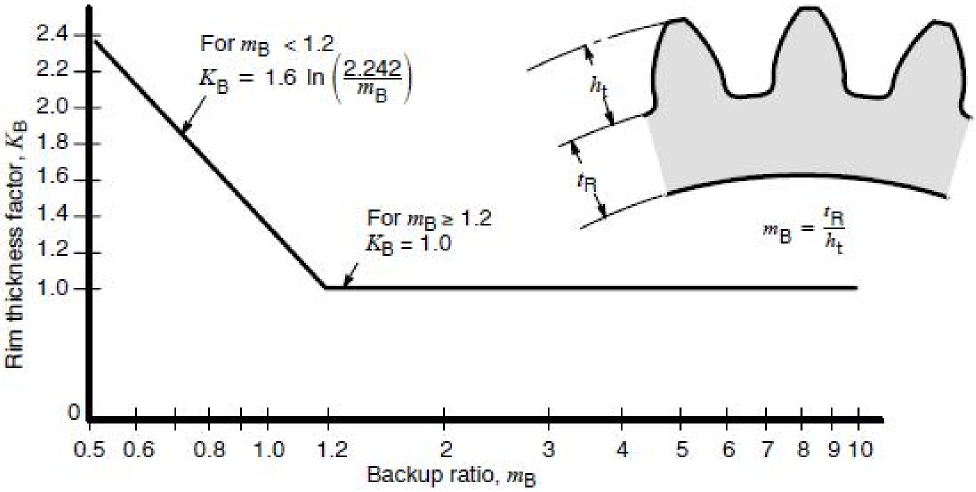


Figure 3.14. Rim thickness factor (ANSI/AGMA 2101 – D04, 2004)

The bending strength geometry factor, Y_J, takes into account the effects of shape of the tooth, worst load position, stress concentration and load sharing between oblique lines of contact in helical gears. Both tangential (bending) and radial (compressive) components of the tooth load are included. This analysis applies to external gears only (ANSI/AGMA 2101 - D04, 2004).

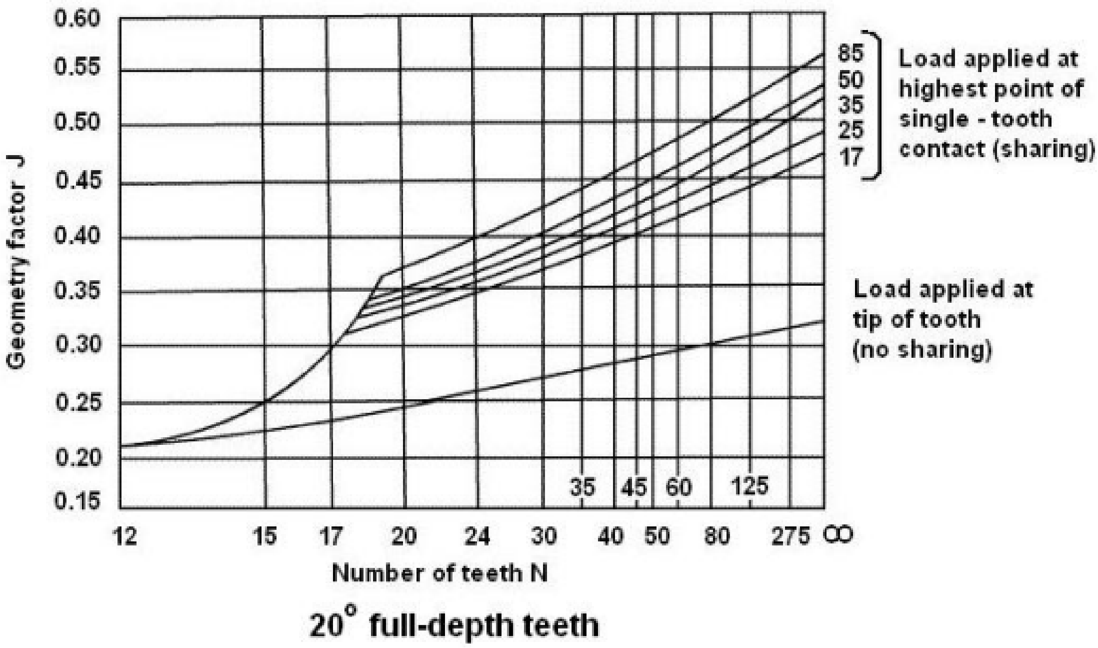


Figure 3.15. Geometry factor Y_J for standard spur gears (ANSI/AGMA 2101 – D04, 2004)

The relation of calculated bending stress number to allowable bending stress number is;

$$\sigma_F \leq \frac{\sigma_{FP} . Y_N}{S_F . Y_\theta . Y_Z} \tag{3.37.}$$

Where

s_{FP}: Allowable bending stress number, N/mm^2

Y_N: Stress cycle factor for bending strength

S_F: Safety factor for bending strength

Y_q: Temperature factor

Y_Z: Reliability factor

The allowable stress numbers, s_{FP}, for gear materials vary with items such as material composition, cleanliness, residual stress, microstructure, quality, heat treatment, and processing practices (ANSI/AGMA 2101-D04, 2004).

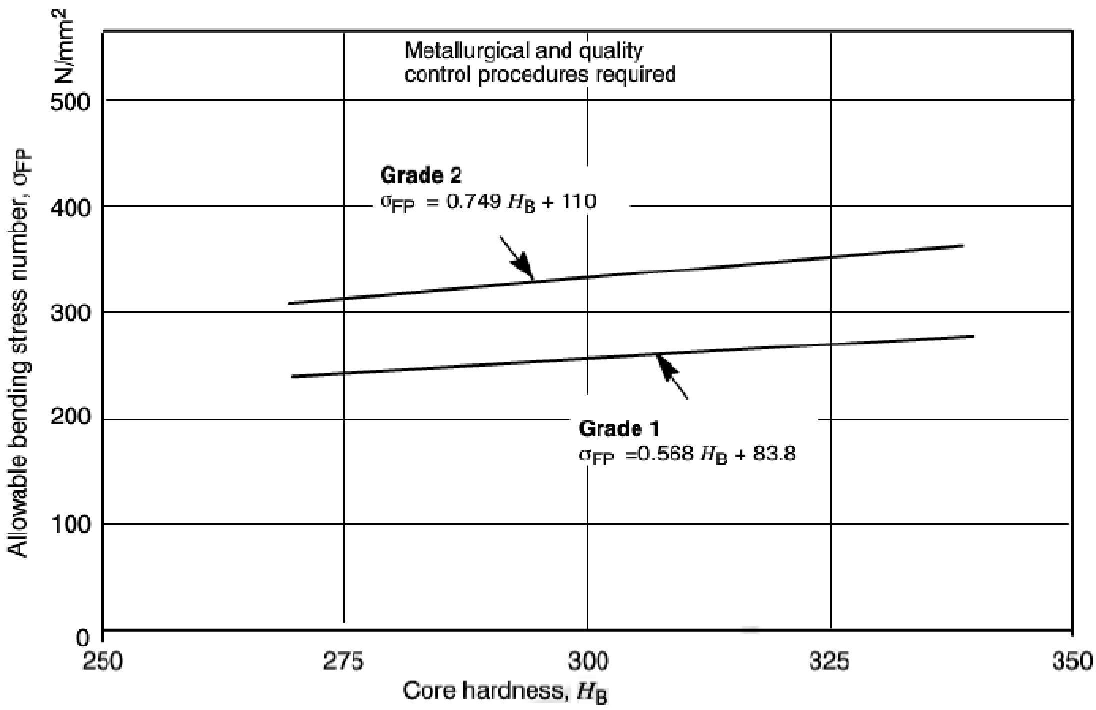


Figure 3.16. Allowable bending stress numbers for nitrided through hardened steel gears (i.e., AISI 4140, AISI 4340), s_{FP} (ANSI/AGMA 2101 – D04, 2004)

The stress cycle factors, Y_N , adjust the allowable stress numbers for the required number of cycles of operation. For the purpose of this standard, N, the number of stress cycles is defined as the number of mesh contacts, under load, of the gear tooth being analyzed (ANSI/AGMA 2101 - D04, 2004).

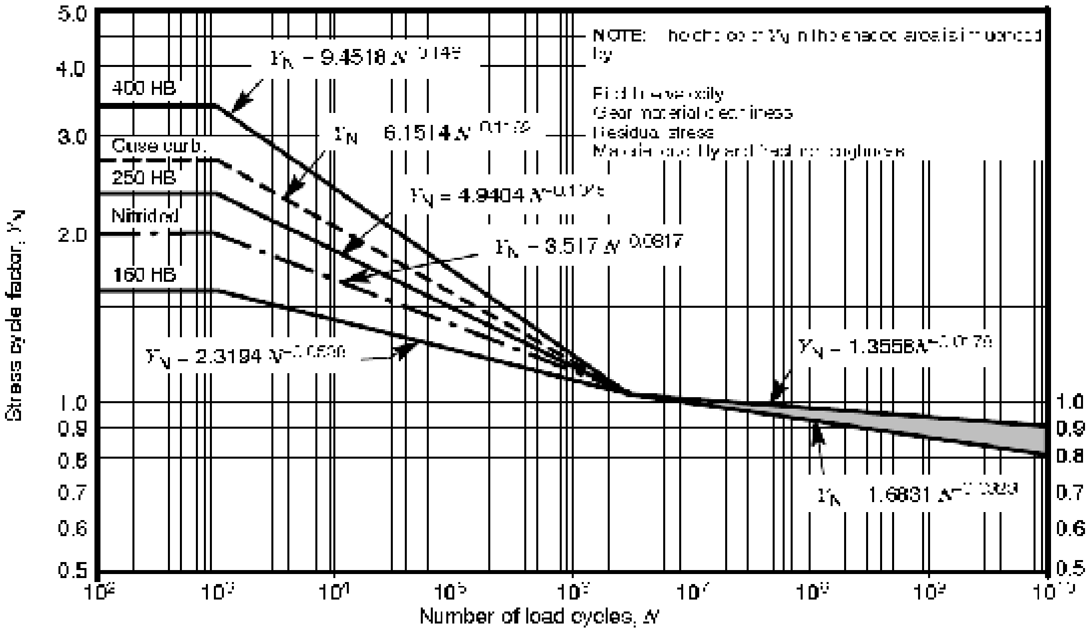


Figure 3.17. Bending strength stress cycle factor, Y_N (ANSI/AGMA 2101 – D04, 2004)

For moderate and low temperature operations the temperature factor, Y_q, is generally taken as unity when gears operate with temperatures of oil or gear blank not exceeding 120°C (ANSI/AGMA 2101 - D04, 2004).

The reliability factors account for the effect of the normal statistical distribution of failures found in materials testing.

Table 3.19 Reliability factors, Y_Z (ANSI/AGMA 2101 – D04, 2004)

Requirements of application	Y_Z [1)]
Fewer than one failure in 10 000	1,50
Fewer than one failure in 1000	1,25
Fewer than one failure in 100	1,00
Fewer than one failure in 10	0,85 [2)]
Fewer than one failure in 2	0,70 [2) 3)]

NOTES
1) Tooth breakage is sometimes considered a greater hazard than pitting. In such cases a greater value of Y_Z is selected for bending.
2) At this value plastic flow might occur rather than pitting.
3) From test data extrapolation.

In ANSI/AGMA 2101 - D04 Standard, the value of design factor of safety, S_H, is not identified certainly. When K_o and Y_Z are used for applying ratings an additional safety factor should be considered to allow for safety and economic risk considerations along with other unquantifiable aspects of the specific design and application (variations in manufacturing, analysis, etc.).The greater the uncertainties or consequences of these considerations, the higher the safety factor should be (ANSI/AGMA 2101 - D04, 2004).

3.2.3. Spur Gear Design Based on Surface Contact Failure

3.2.3.1 Design Approach Using Mechanical Engineering Design 1st Metric Edition

Failure of the surfaces of gear teeth, generally called as wear. Pitting is a surface fatigue failure due to many repetitions of high contact stresses. Calculation of face width relies on the same procedure as in bending fatigue failure, surface compressive stress should be equal or less than the surface fatigue stress (Sgihley J.E., 1985).

$$\sigma_H = -C_p . \sqrt{\frac{W_t}{C_v.F.d_p.I}} \tag{3.38.}$$

Where

s_H: Surface compressive stress, MPa
C_p: Elastic coefficient, $(MPa)^{1/2}$
W_t: Tangential component of load, in N
C_v: Velocity factor
F: Face width of gear tooth, in mm
d_p: Pitch diameter, in mm
I: Geometry factor

The calculation of W_t is executed using Equations 3.2 and 3.3. And the velocity factor, C_v, equals to K_v as described in Equation 3.4.

Elastic coefficient, C_p, is determined from Table 3.20 according to the material of both pinion and gear.

Table 3.20. Values of the elastic coefficient C_p for spur and helical gears with non-localized contact and for n = 0,30 (Shigley J.E., 1985)

Pinion	Modulus of elasticity, E, GPa	Gear					
		Steel	Malleable iron	Nodular iron	Cast iron	Aluminum bronze	Tin bronze
Steel	200	191	181	179	174	162	158
Malleable iron	170	181	174	172	168	158	154
Nodular iron	170	179	172	170	166	156	152
Cast iron	150	174	168	166	163	154	149
Aluminum bronze	120	162	158	156	154	145	141
Tin bronze	110	158	154	152	149	141	137

Geometry factor, I, is calculated as;

$$I=\frac{\cos\emptyset.\sin\emptyset}{2}\cdot\frac{m_G}{m_G+1} \tag{3.39.}$$

Where m_G is the speed ratio as;

$$m_G=\frac{N_G}{N_P}=\frac{d_G}{d_P} \tag{3.40.}$$

The surface fatigue strength for steels is given as;

$$S_C=2,76.(HB)-70MPa \tag{3.41.}$$

Where HB is the Brinell hardness of the softer of the two contacting surfaces. The value given by Equation 3.41 corresponds to a life of 10^8 stress application (Shigley J.E., 1985).

The AGMA recommends that this contact fatigue strength must be modified in a manner quite similar to that used for the bending endurance limit (Sgihley J.E., 1985). The equation is;

$$S_H = \frac{C_L.C_H}{C_T.C_R}.S_C \tag{3.42}$$

Where

S_H: Corrected fatigue strength, or Hertzian strength

C_L: Life factor, Table 3.21

C_H: Hardness ratio factor; use 1,0 for spur gears

C_T: Temperature factor; use 1,0 for temperatures less than 120°C

C_R: Reliability factor, Table 3.21

The life modification factor, C_L, is used to increase the strength when the gear is to be used for short periods of time.

Table 3.21. Life and reliability modification factors (Shigley J.E., 1985)

Cycles of life	Life factor, C_L	Reliability, R	Reliability factor, C_R
10^4	1,5	Up to 0,99	0,8
10^5	1,3	0,99 to 0,999	1,0
10^6	1,1	0,999 up	1,25 up
10^8 up	1,0		

The hardness ratio factor, C_H, was included by AGMA, to account for differences in strength due to the fact that one of the mating gears might be softer than the other. However, for spur gears, use $C_H = 1,0$.

The AGMA makes no recommendations on values to use for the temperature factor C_T when the temperature exceeds 120°C, except to imply that a value $C_T > 1,0$ should probably be used. To a large extent this will depend upon the temperature limitations of the lubricant used, since the materials should withstand larger temperatures (Sgihley J.E., 1985). Use Equation 3.6 to determine C_T.

Design factor of safety to guard against surface failures should be selected. The AGMA uses C_o and C_m to designate the overload and load distribution factors, but their values are the same as those for K_o and K_m. These factors should be used in the numerator of Equation 3.38 as load multiplying factor. Designate the permissible transmitted load, $W_{t,p}$ as;

$$W_{t,p} = n_G . W_t \tag{3.43.}$$

Where n_G is calculated as before using Equation 3.8.
Equation 3.38 can now be written as;

$$S_H = C_p \sqrt{\frac{W_{t,p}}{C_v.F.d_P.I}} \tag{3.44.}$$

3.2.3.2. Design Approach Using Shigley's Mechanical Engineering Design 9th Edition

In this approach a surface failure occurs when the significant contact stress equals or exceeds the surface endurance strength.

$$\sigma_C = -C_p . \left[\frac{K_v.W^t}{F.\cos\emptyset} . \left(\frac{1}{r_1} + \frac{1}{r_2}\right)\right]^{1/2} \tag{3.45.}$$

Where

C_p:	Elastic coefficient and negative sign means s_C is a compressive stress
r_1 and r_2:	Instantaneous values of the radii of curvature on the pinion and gear tooth profiles, respectively, at the point of contact
W^t:	Tangential component of load, in N
K_v:	Velocity factor
F:	Face width, in mm

Elastic coefficient, C_p, is calculated as;

$$C_p = \left[\frac{1}{\pi.\left(\frac{1-v_P^2}{E_P}+\frac{1-v_G^2}{E_G}\right)}\right]^{1/2} \quad (3.46.)$$

Where

ν_P and ν_G are the poisson's ratio and E_P and E_G are the modulus of elasticity of pinion and gear materials respectively.

$$r_1 = \frac{d_P.\sin\emptyset}{2} \qquad r_2 = \frac{d_G.\sin\emptyset}{2} \quad (3.47.)$$

Where

$\emptyset$: Pressure angle and d_P and d_G are the pitch diameters of the pinion and gear, respectively.

The velocity factor, K_v, is calculated from Equation 3.12.

Surface endurance strength is determined by as a longstanding correlation in steels between S_C and H_B at 10^8 cycles is;

$$(S_C)_{10^8} = (2{,}76).H_B - 70 \text{ MPa} \quad (3.48.)$$

In order to find the value of face width, Equation 3.45 is equaled to surface endurance strength Equation 3.48 by considering a design factor of safety.

3.2.3.3. Design Approach Using Fundamentals of Machine Component Design 5th Edition

The approach given by Juvinall and Marshek recommends that gear tooth surface fatigue stress have to be equal or less than gear tooth surface fatigue strength by considering a certain value of design factor of safety. Gear tooth surface fatigue stress is calculated as;

$$\sigma_H=C_p.\sqrt{\frac{F_t}{b.d_p.I}.K_v.K_o.K_m} \quad (3.49.)$$

C_p: Commonly called the elastic coefficient in the unit of $\sqrt{MPa}$ and its value is read from table below.

Table 3.22. Values of Elastic Coefficient C_p for Spur Gears in $\sqrt{MPa}$ (Juvinall R.C., and Marshek K.M., 2011)

Pinion Material (n = 0,30 in All Cases)	Gear Material			
	Steel	Cast Iron	Aluminum Bronze	Tin Bronze
Steel, E = 207 GPa	191	166	162	158
Cast iron, E = 131 GPa	166	149	149	145
Aluminum bronze, E = 121 GPa	162	149	145	141
Tin bronze, E = 110 GPa	158	145	141	137

I: Commonly called the geometry factor;

$$I=\frac{\sin\emptyset.\cos\emptyset}{2}.\frac{R}{R+1} \quad (3.50.)$$

Here R is the ratio of gear and pinion diameters,

$$R=\frac{d_g}{d_p} \quad (3.51.)$$

Gear tooth surface fatigue strength is calculated as;

$$S_H=S_{fe}.C_{Li}.C_R \quad (3.52.)$$

Where

S_{fe}: Surface fatigue strength determined from Table 3.23.

Table 3.23. Surface Fatigue Strength S_{fe}, for Use with Metallic Spur Gears (10^7-Cycle Life, 99 Percent Reliability, Temperature <250°F) (Juvinall R.C., and Marshek K.M., 2011)

Material	S_{fe} (ksi)	S_{fe} (MPa)
Steel	0,4 (Bhn) - 10 ksi	28 (Bhn) - 69 MPa
Nodular iron	0,95[0,4 (Bhn) – 10 ksi]	0,95 [28 (Bhn) - 69 MPa]
Cast iron, grade 20	55	379
grade 30	70	482
grade 40	8	551
Tin bronze AGMA 2C (11 percent tin)	30	207
Aluminum bronze (ASTM B 148 - 52) (Alloy 9C - H.T.)	65	448

C_{Li}: Life factor, see Figure 3.17. For fatigue lives other than 10^7 cycles, multiply the values of S_{fe} by a life factor.

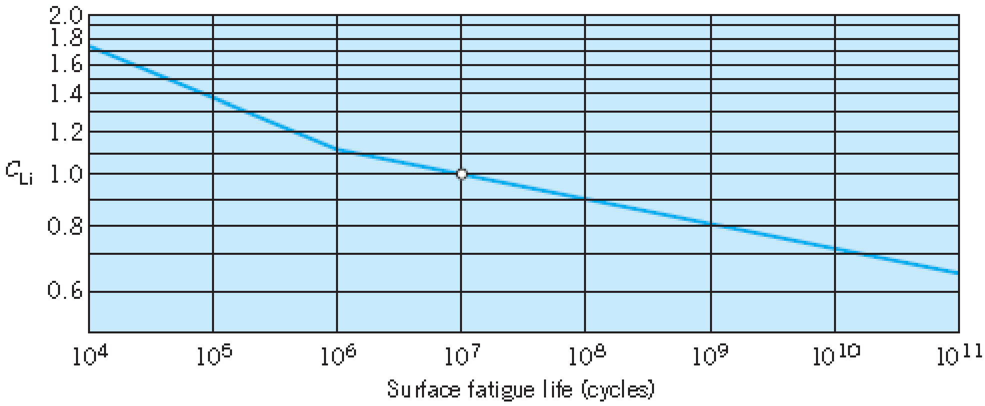


Figure 3.18. Values of C_{Li} for steel gears (general shape of surface fatigue S–Ncurve) (Juvinall R.C., and Marshek K.M., 2011)

C_R: Reliability factor determined from Table 3.24.

Table 3.24. Value of reliability factor (Juvinall R.C., and Marshek K.M., 2011)

Reliability (%)	**C_R**
50	1,25
99	1,00
99,9	0,80

Now by equalizing the Equations 3.49 and 3.52 with an addition of a design factor of safety, face width, b, can be determined.

3.2.3.4. Design Approach Using ISO Standards 9085:2002

ISO Standards 9085:2002 also provides gear design formula based on the surface contact. The calculation of surface durability is based on surface contact stress, s_H, at the pitch point or at the inner (lowest) point of single pair tooth contact. The higher of the two values obtained is used to determine capacity. The values of s_H and the permissible contact stress, s_{HP}, shall be calculated separately for wheel and pinion; s_H shall be less than or equal to s_{HP} (ISO 9085:2002, 2002).

Determination of contact stress, σ_H, for the pinion is given in the standard of ISO 9085:2002 as follow;

$$\sigma_H = Z_B.\sigma_{H0}\sqrt{K_A.K_V.K_{H\beta}.K_{H\alpha}} \leq \sigma_{HP} \quad (3.53.)$$

With

$$\sigma_{H0} = Z_H.Z_E.Z_\varepsilon.Z_\beta.\sqrt{\frac{F_t}{d_1.b_H}.\frac{u+1}{u}} \quad (3.54.)$$

Where

σ_H: Contact stress

σ_{H0}: Nominal contact stress at the pitch point

σ_{HP}: Permissible contact stress

Z_B: Single pair tooth contact factor for the pinion

Z_H: Zone factor

Z_E: Elasticity factor

Z_ε: Elastic coefficient for pitting

Z_β: Helix angle factor for pitting

u: Gear ratio

Single pair tooth contact factor, Z_B, is used to transform the contact stress at the pitch point of spur gears to the contact stress at the inner (lowest) limit of single pair tooth contact of the pinion.

$$M_1 = \frac{\tan\alpha_{wt}}{\sqrt{\left[\sqrt{\frac{d_{a1}^2}{d_{b1}^2}-1}-\frac{2.n}{z_1}\right].\left[\sqrt{\frac{d_{a2}^2}{d_{b2}^2}-1}-(\varepsilon_a-1)\frac{2.)}{z_2}\right]}} \quad (3.55.)$$

Where ε_a is the transverse contact ratio;

$$\varepsilon_a = g_a / p_{bt} \quad (3.56.)$$

$g_a = \frac{1}{2}\left[\sqrt{d_{a1}^2-d_{b1}^2}+\sqrt{d_{a2}^2-d_{b2}^2}\right]$ -a.sinα_{wt} , where a is the center distance

$p_{bt}=m_t$.t theα_t

If $M_1>1$ then $Z_B=M_1$ If $M_1\leq1$ then $Z_B=1,0$

Application factor, K_A, is determined from Table 3.14.

The elasticity factor, Z_E, takes into account the influence of the material properties E (modulus of elasticity) and ν (Poisson’s ratio) on the contact stress. Numerical values are given in Table 3.25 below.

Table 3.25. Elastic factor (ISO 9085:2002, 2002)

Wheel 1			Wheel 2			Z_E $\sqrt{N/mm^2}$
Material[a]	**Modulus of elasticity N/mm^2**	**Poisson's ratio** n	**Material[a]**	**Modulus of elasticity N/mm^2**	**Poisson's ratio** n	
St, V, Eh, NT (nitr.), NV (nitr.), NV (nitrocar.)	206 000	0,3	St, V, Eh, NT (nitr.), NV (nitr.), NV (nitrocar.)	206 000	0,3	189,8
			St (cast)	202 000		188,9
			GGG (perl.,bai., ferr.)	173 000		181,4
			GTS (perl.)	170 000		180,5
			GG	126 000 to 118 000		165,4 to 162,0
St (cast)	202 000		St (cast)	202 000		188,0
			GGG (perl.,bai., ferr.)	173 000		180,5
			GTS (perl.)	170 000		179,7
			GG	118 000		181,4
GGG (perl.,bai., ferr.)	173 000		GGG (perl.,bai., ferr.)	173 000		173,9
			GTS (perl.)	170 000		173,2
			GG	118 000		156,6
GTS (perl.)	170 000		GTS (perl.)	170 000		172,4
			GG	118 000		156,1
GG	126 000 to 118 000		GG	118 000		146,0 to 143,7

[a] See Appendix D for an explanation of the abbreviations used (ISO 6336-1:2006).

The contact ratio factor, Z_ε, equals to 1 for spur gears.

$$Z_\epsilon=\sqrt{\frac{4-e_a}{3}} \tag{3.57.}$$

Where ε_a is the transverse contact ratio, calculated using Equation 3.56.

Z_ε = 1,0 may be chosen for spur gears having a contact ratio of less than 2,0.

Helix angle factor, Z_β takes into account of the influence on surface stress of the helix angle.

$Z_\beta=\sqrt{\cos\beta}$

Since $\beta = 0$ for spur gears, $Z_\beta = 1,0$.

The zone factor Z_H, accounts for the influence on Hertzian pressure of tooth flank curvature at the pitch point and transforms the tangential force at the reference cylinder to normal force at the pitch cylinder.

$$Z_H = \sqrt{\frac{2.\cos\beta_b.\cos\alpha_{wt}}{\cos\alpha_t^2.\sin\alpha_{wt}}} \qquad (3.58.)$$

Where

β_b: Base helix angle

α_{wt}: Transverse pressure angle at the pitch cylinder

α_t: Transverse pressure angle

Determination of permissible contact stress, σ_{HP}, the method B of ISO 6336-2:1996 is used in this International Standard.

$$\sigma_{HP}=\frac{\sigma_{Hlim}.Z_{NT}}{S_{Hmin}}.Z_L.Z_V.Z_R.Z_W.Z_X \qquad (3.59.)$$

Where

$s_{H\ lim}$: Allowable stress number for pitting

Z_{NT}: Life factor for contact stress for reference test conditions

Z_L: Lubricant factor

Z_V: Speed factor

Z_R: Roughness factor affecting surface durability for ISO Standard

Z_W: Work hardening factor pitting

Z_X: Size factor pitting

$S_{H\ min}$: Minimum safety factor for pitting

Allowable stress number for pitting is determined according to Table 3.16 with the aid of following formula;

$$\sigma_{Hlim}=A.x+B \quad (3.60.)$$

The life factor, Z_{NT}, method B of ISO 6336-3:1996 is used in this International Standard.

Table 3.26. Determination of life factor (ISO 9085:2002, 2002)

Material[a]	**Number of load cycles**	**Life factor Z_{NT}**
St, St (cast), V, GGG (perl. Bain.), Eh, IF Only when a certain degree of pitting is permissible	$N_L \leq 6$ ´ 10^5 (static)	1,6
	$N_L = 10^7$	1,3
	$N_L = 10^7$ (reference)	1,0
	$N_L = 10^{10}$	ME, MX :1,0[b]
		MQ : 0,92
		ML : 0,85
St, St (cast), V, GGG (perl. Bain.), Eh, IF No pitting is permissible	$N_L \leq 10^5$ (static)	1,6
	$N_L = 5$ ´ 10^7 (reference)	1,0
	$N_L = 10^{10}$	ME :1,0[b]
		MQ : 0,92
		ML : 0,85
GG, GGG (ferr.), NT (nitr.), NV (nitr.)	$N_L \leq 10^5$ (static)	1,3
	$N_L = 2$ ´ 10^6 (reference)	1,0
	$N_L = 10^{10}$	ME :1,0[b]
		MQ : 0,92
		ML : 0,85
NV (nitrocar.)	$N_L \leq 10^5$ (static)	1,1
	$N_L = 2$ ´ 10^6 (reference)	1,0
	$N_L = 10^{10}$	ME :1,0[b]
		MQ : 0,92
		ML : 0,85
[a] See Appendix D for an explanation of the abbreviations used (ISO 6336-1:2006).		
[b] Optimum lubrication, manufacturing and experience supposed.		

In ISO 6336-2:1996 standard, influences on lubrication film formation has been taken by using following factors; Z_L, accounts for the influence of nominal viscosity of the lubricant, Z_V, for the influence of tooth flank velocities and Z_R, for the influence of surface roughness on the formation of the lubricant film in the contact zone. Method C of the ISO 6336-2:1996 is used in this International Standard.

For gears which are hobbed, shaped or planed, or which do not meet the following three conditions;

$Z_L.Z_V.Z_R = 0,85$

For gears with lapped, ground or shaved teeth and mean relative peak to valley, $Rz_{10} > 4$ μm ;

$Z_L.Z_V.Z_R = 0,92$

For gear pairs in which one gear is hobbed, shaped or planed and the mating gear is ground or shaved with $Rz_{10} \leq 4$ μm ;

$Z_L.Z_V.Z_R = 0,92$

For ground and shaved gearing with $Rz_{10} \leq 4$ μm ;

$Z_L.Z_V.Z_R = 1,0$

The work hardening factor, Z_W takes into account of the increased surface durability due to meshing a steel wheel (structural steel, through - hardened steel) with a pinion which is significantly (≅200 HB or more) harder than the wheel and with smooth tooth flanks ($Rz \leq 6$ μm, otherwise effects of wear are not covered by this International Standard). Method B of ISO 6336-2:1996 is applied, as follows;

If HB < 130 then

$Z_W = 1,2$

If $130 \leq HB \leq 470$ then

$$Z_W = 1,2 - \frac{HB-130}{1700} \quad (3.61.)$$

If HB > 470 then

$Z_W = 1,0$

Where HB is the Brinell hardness of the tooth flanks of the softer gear of pair.

Size factor, Z_X, is affected from material quality (furnace charge, cleanliness, forging), heat treatment, depth of hardening, distribution of hardening, radius of flank curvature and module in the case of surface hardening, depth of hardened layer relative to the size of teeth (core supporting effect).

For through hardened gears and for surface hardened gears with adequate case depth relative to tooth size and radius of relative curvature, Z_X , is taken to be 1,0.

ISO Standards suggest a minimum design factor for pitting, $S_{H\ min}$, shall be applied as 1,0 if not otherwise should be agreed between manufacturer and user. But as mentioned before minimum safety factor has been taken as a value of 2,1.

3.2.3.5. Design Approach Using ANSI/AGMA 2101 - D04 Standards

In ANSI/AGMA Standard, the contact stress number have to be equal or less than the allowable contact stress number. Contact stress number is calculated as;

$$\sigma_H = Z_E \cdot \sqrt{F_t . K_o . K_v . K_S \cdot \frac{K_H}{d_{w1} . b} \cdot \frac{Z_R}{Z_I}} \tag{3.62.}$$

Where

s_H: Contact stress number, N/mm^2

Z_E: Elastic coefficient, $[N/mm^2]^{0.5}$

F_t: Transmitted tangential load, N

K_o: Overload factor

K_v: Dynamic factor

K_S: Size factor

K_H: Load distribution factor

Z_R: Surface condition factor pitting resistance

Z_I: Geometry factor for pitting resistance

d_{w1}: Operating pitch diameter of pinion, mm.

$$d_{w1}=\frac{2.C}{m_G+1} \text{ for external gears}$$

Where

C is operating center distance in mm and m_G is gear ratio (never less than 1,0).

The Elastic Coefficient, ZE is defined by the following equation;

$$Z_E = \sqrt{\frac{1}{\pi i\left[\left(\frac{1iv_1^2}{E_1}\right)+\left(\frac{1iv_2^2}{E_2}\right)\right]}} \qquad (3.63.)$$

Where

Z_E: Elastic coefficient, $[N/mm^2]^{0.5}$

n_1 and n_2: Poisson's ratio for pinion and gear, respectively

E_1 and E_2: Modulus of elasticity for pinion and gear, respectively, N/mm^2

The factors, K_o, K_v, K_S, K_H, has been defined before as in section 3.2.2.5. The same steps should be carried out for determining these factors.

The surface condition factor, Z_R, is used only in the pitting resistance formula, depends on surface finish as affected by, but not limited to, cutting, shaving, lapping, grinding, shot peening, residual stress, and plasticity effects (work hardening). The surface condition factor can be taken as unity if the appropriate surface condition is provided (ANSI/AGMA 2101-D04, 2004).

The pitting resistance geometry factor, Z_I, evaluates the radii of curvature of the contacting tooth profiles based on tooth geometry. These radii are used to evaluate the Hertzian contact stress in the tooth flank. Effects of modified tooth proportions and load sharing are considered. AGMA 908-B89 (1989) Standard provides the pitting resistance geometry factor as follows;

$$Z_I = \frac{\cos\emptyset_r . C_y^2}{\left(\frac{1}{\rho_1} \pm \frac{1}{\rho_2}\right) . d . m_N} \tag{3.64.}$$

Where

$\emptyset_r$: Operating transverse pressure angle

C_y: Helical overlap factor

d: Pinion operating pitch diameter

r 1: Radius of curvature of pinion profile at point of contact stress calculation

r 2: Radius of curvature of pinion profile at point of contact stress calculation

m_N: Load sharing factor = 1 for spur gears

$$\rho_1 = \left(R_{m1}^2 - R_{b1}^2\right)^{0,5}$$

(3.65.)

$$\rho_2 = C_6 + \rho_1$$

(3.66.)

Where

R_{m1}: mean radius of pinion

R_{b1}: base radius, pinion

$$R_{m1} = \frac{1}{2} . [R_{o1} + (C_r - R_{o2})]$$

(3.67.)

Where

R_{o1}: addendum radius, pinion

R_{o2}: addendum radius, gear

C_6: sixth distance along line of action

$$C_6 = C_r . \sin\phi_r \tag{3.68.}$$

ϕ_r: operating transverse pressure angle

$$\phi_r = \cos^{-1}\left(\frac{R_{b2}+R_{b1}}{C_r}\right) \tag{3.69.}$$

Allowable contact stress number is calculated as follow;

$$\sigma_H \leq \frac{\sigma_{HP}}{S_H} . \frac{Z_N}{Y_\theta} . \frac{Z_W}{Y_Z}$$

Where

σ_{HP}: Allowable contact stress number, N/mm²

Z_N: Stress cycle factor for pitting resistance

Z_W: Hardness ratio factor for pitting resistance

S_H: Safety factor for pitting

Y_θ: Temperature factor

Y_Z: Reliability factor

The hardness ratio factor, Z_W, depends upon gear ratio, surface finish of pinion, hardness of pinion and gear.

$$Z_W = 1{,}0 + A . (m_G - 1{,}0) \tag{3.70.}$$

$$A = 0{,}00898 \times \left[\frac{H_{BP}}{H_{BG}}\right] - 0{,}00829$$

for $\frac{H_{BP}}{H_{BG}} < 1{,}2$, $A = 0{,}0$

Temperature and reliability factors, Y_q, and Y_Z, has been mentioned previously in section 3.2.2.5.

Safety factor, S_H, has been taken as 2,1 which is the same with the other approaches.

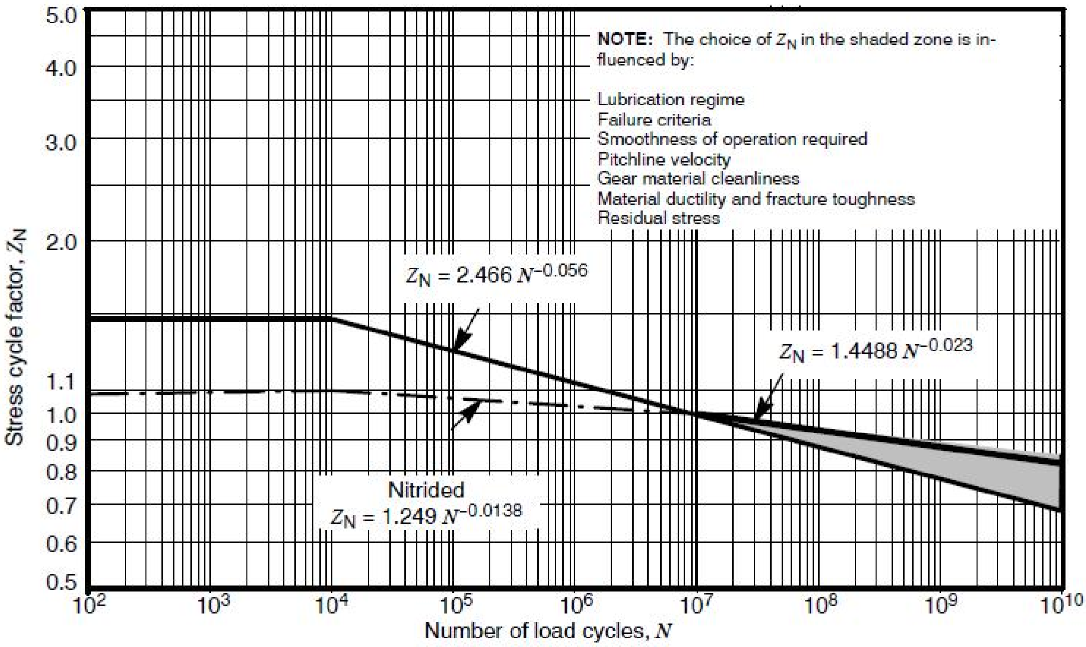


Figure 3.19. Pitting resistance stress cycle factor, Z_N (ANSI/AGMA 2101-D04, 2004)

After determining the factors that affect the contact stress number and allowable contact stress, face width, b, can be determined by arranging the stress formula, Equation 3.62.

3.3. Development of Microsoft Excel Pages

Module selection and face width determination require iterations as described in Figure 3.2. And depending on the experiences of designer, iterations takes considerable calculation time durations. For this reason, all the determinations that referred in Section 3.2.2 and 3.2.3, have been performed by using Microsoft Excel pages. This method has allowed simplicity for designing a spur gear iteratively since it needs complicated determinations. Therefore loss or gain in volume or in selection of material type or stress related performance has been seen easily on excel pages by changing the parameters.

The excel pages have been carried out with a systematic way. As it is seen in Figure 3.20, a gear design includes input parameters and design variables in order to find the design outputs that are the suitable module (m) and the appropriate face width (F). Both m and F are the most important design parameters.

Input parameters have been defined before starting the gear design as it is given in Table 3.27 and specified in excel pages. These input parameters can then be changed according to the requirements of users or operating conditions if it is needed.

Table 3.27. Selected input parameters for the design

Input Parameters
Pressure angle, Æ
Type of gear profile
Input speed of a power source, rpm
Number of life cycles, N
Design factor of safety, n_d
Reliability, %
Operating temperature, T
Quality number for gear
Material properties of gear pair
Working characteristics of driving and driven machines
Selected transmitted power range, kW
Selected Gear speed ratio range, m_G

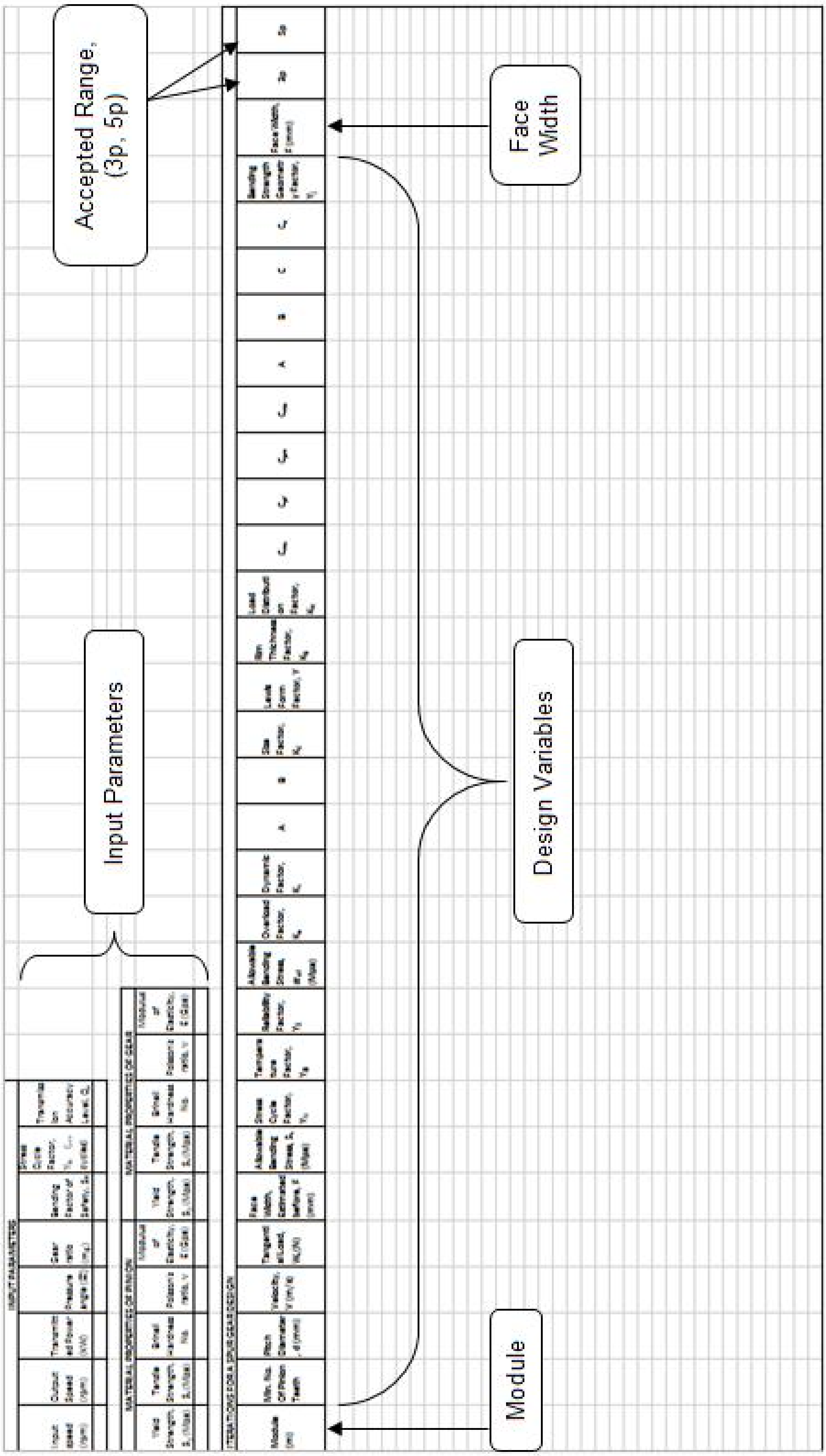


Figure 3.20. Page format of a Microsoft Excel page for an involute spur gear design

The layout of defined input parameters in excel pages are shown in Figure 3.21. As it is seen from the figure, the input parameters cover the operating conditions, material properties of a pair of gear. It also gives an information about gear tooth profile. Figure 3.21 shows the input parameters that are entered into the excel pages prepared for the ANSI/AGMA 2101-D04 Standards (2004). Although these input parameters have been kept identical for the design approaches, there have been slight differences for the values of input parameters. This is because design variables are taken into account in different ways for each of the design approaches.

INPUT PARAMETERS							
Input speed, rpm	Output Speed, rpm	Transmitted Power, kW	Pressure angle $\varnothing$	Gear speed ratio, m_G	Bending Factor of Safety, S_F	Stress Cycle Factor, Y_N (... cycles)	Transmission Accuracy Level, Q_v
1200	1200	10	20	1	2,1	1E+08	9

MATERIAL PROPERTIES OF PINION					MATERIAL PROPERTIES OF GEAR				
Yield Strength, S_y (Mpa)	Tensile Strength, S_{ut} (Mpa)	Brinell Hardness No.	Poisson's ratio, ν	Modulus of Elasticity, E (Gpa)	Yield Strength, S_y (Mpa)	Tensile Strength, S_{ut} (Mpa)	Brinell Hardness No.	Poisson's ratio, ν	Modulus of Elasticity, E (Gpa)
1140	1250	370	0,3	200	621	827	400	0,3	170

OPERATING CONDITIONS		
Temperature Factor, Y_θ	**Reliability Factor, Y_Z**	**Overload Factor, K_o**
1	1,25	1

	input parameters that affect speed ratio
	input parameter for power transmission
	input parameters for service conditions

Figure 3.21. Input parameters that represented on excel pages

The approach of using Excel pages enabled to obtain the results in a very short time for the various selected speed ratios and for the selected power transmission ranges.

3.4. Development of Finite Element Method (FEM)

The Finite Element Method (FEM) is a numerical analysis technique for obtaining approximate solutions to a wide variety of engineering problems. For many engineering problems which consist of complex mathematical models such as designing a gear, it is not always possible to obtain analytical solutions. For this reason numerical methods provide approximate but acceptable solutions. As mentioned in Chapter 2, in this work, numerical results of FEM is compared with the analytical results of the selected gear design approaches. This is mainly used to verify the analytical results, and to select the best gear design approach which is used for various comparisons to obtain more refined results.

In this thesis work, module and face width have been found by analytical methods iteratively with the aid of excel pages and a numerical “*Analysis System”* (ANSYS) Workbench 14.0 has been used to compare the analytical results with numerical solutions. The software ANSYS needs a structural model to execute the analysis. So by using the design parameters, and the obtained module and face width through the iterations, a 3D model of pinion was created on “*Computer Aided Three Dimensional Interactive Application”* (CATIA, V5 R20).

Structural analysis requires three steps generally: preprocessing, solver and post processing. In preprocessing, the geometry of structure is made and creating mesh elements, solver is the defining of boundary conditions and lastly in post processing analysis results are obtained.

In the following chapter, the use of CATIA and ANSYS softwares have been given including the design results.

Since the ANSYS software analyzes the gear stresses, gear bending stress has been determined numerically considering the final design results of module and face width. Spur gears that have been designed for 1:1 speed ratio at 10 kW power transmission have been modelled using the same design input parameters. The results obtained and provided in the following section gave highest module values at 1:1 speed ratio. As a result of this 1:1 speed ratio was selected as the most critical ratio for the module. The selection of power is not as straight forward as speed ratio. The most common power range in many industrial applications is 1 to 10 kW. Therefore 10 kW is selected as the power input. It is believed that these input values may allow to obtain suitable conclusions. The FEM results have been obtained for the five gear design approaches based on bending fatigue failure criteria.

3.5. Summary

The formulas in the gear design approaches given in the previous sections were rewritten to obtain the face width (F). The obtained face width equations for each type of design approaches have been represented in Table 3.28 based on bending stress and in Table 3.29 based on surface contact stress. As it is seen from the tables, there are significant differences when comparing the different type of design approaches. Each face width equation depends on some design variables that are completely or totally differ to each other. These equations are then used in the Excel pages together with the all inputs.

Table 3.28. Face width equations of the design approaches based on bending fatigue stress failure criteria

Design Approaches	Face Width
Mechanical Engineering Design 1st Metric Ed.	$F=\dfrac{W_t.n_d.K_o.K_m}{K_v.m.J.k_a.k_b.k_c.k_d.k_e.k_f.S_e^{'}}$
Shigley's Mechanical Engineering Design 9th Ed.	$F=\dfrac{K_v.W^t.n_d}{m.Y.k_a.k_b.k_c.k_d.k_e.k_f.S_e^{'}}$
Fundamental Of Machine Component Design 5th Ed.	$b=\dfrac{n_d.F_t}{m.J.S_n^{'}.C_L.C_G.C_S.k_r.k_t.k_{ms}}.K_V.K_o.K_m$
ISO 6336 Standards	$b=\dfrac{S_{Fmin}.F_t}{\sigma_{F_{lim}}.Y_{ST}.Y_{NT}.Y_{\delta\ rel\ T}.Y_{R\ rel\ T}.Y_X.m_n}.Y_F.Y_S.Y_b.Y_B.Y_{DT}.\ K_A.K_V.K_{Fb}.K_{Fa}$
ANSI/AGMA 2101 - D04 Standards	$b=\dfrac{S_F.F_t}{\sigma_{FP}.m_t.Y_J}.\dfrac{Y_\theta.Y_Z}{Y_N}.K_O.K_V.K_S.K_H.K_B$

Table 3.29. Face width equations of the design approaches based on contact fatigue stress failure criteria

Design Approaches	Face Width
Mechanical Engineering Design 1st Metric Ed.	$F=\left(\frac{C_p}{S_C}\right)^2.\left(\frac{C_T.C_R}{C_L.C_H}\right)^2.\frac{W_t.n_d.C_o.C_m}{C_v.d_p.I}$
Shigley's Mechanical Engineering Design 9th Ed.	$F=\left(\frac{C_p}{S_C}\right)^2.\frac{K_v.W^t.n_d}{\cos\emptyset}.\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$
Fundamental Of Machine Component Design 5th Ed.	$b=\left(\frac{C_p}{S_{fe}}\right)^2.\frac{1}{C_{Li}^2.C_R^2}.\frac{F_t.K_v.K_o.K_m.n_d^2}{d_p.I}$
ISO 9085:2002 Standards	$b_H=\left(\frac{Z_E}{\sigma_{H\ lim}}\right)^2.\left(\frac{Z_B.Z_H.Z_\varepsilon.Z_\beta.}{Z_{NT}.Z_L.Z_V.Z_R.Z_W.Z_X}\right)^2.\left(\frac{u+1}{u}\right).\frac{F_t}{d_1}.K_A.K_V.K_{H\beta}.K_{H\alpha}.S_H^2$
ANSI/AGMA 2101 - D04 Standards	$b=\left(\frac{Z_E}{\sigma_H}\right)^2.\left(\frac{Y_\theta.Y_Z}{Z_N.Z_W}\right)^2.\left(\frac{Z_R}{Z_I}\right).\frac{F_t.K_o.K_v.K_S.K_H.S_H^2}{d_{w1}}$

When the above tables are observed it is seen that both “F” and “b” are used for the face width. However “F” has been used for the face width in the following chapters.

4. RESULTS AND DISCUSSIONS

When designing gears, the most important design parameters are module and face width. As mentioned in Chapter 3, these have been determined considering the gear stresses called as bending stress and surface contact stress by using five different type of design approaches, given by Shigley J.E. (1985), Budynas R.G. and Nisbett J.K. (2011), Juvinall R.C. and Marshek K.M. (2011), ANSI/AGMA 2101-D04 Standards (2004), and ISO Standards of 6336 Part 1-3 (2006), Part 5 (2003), Part 6 (2004), and ISO 9085:2002 (2002).

For the selected 5 approaches, equations for face width "F" based on bending stress and face width "F" based on surface contact stress has been obtained considering the five types of gear design approaches or formulations and given in Table 3.28 and 3.29 respectively.

Figure 3.2 in Chapter 3 has also described the iterations needed for proper module selection and face width determination. Before starting the iterations, geometrical criterions, operating conditions and material properties for a pair of gear have been defined as input parameters. While the iterations are carried out, all the input parameters have been kept constant. Table 4.1 shows the input parameters with their values that considered in this study, and they have been kept identical for the five types of design approaches. A fair comparison between the design approaches were obtained by keeping input parameters identical throughout the study. After determining the input parameters that are kept constant for all of the gear designs, iterations for proper module selection were made by determining design variables that affect the failure stresses of material strength. The calculations were carried out until face width, F, is in between 3p and 5p that is considered to be the accepted range.

Table 4.1. Value of The selected Input parameters for the design

Input Parameters	Value
Pressure angle, Æ	20°
Type of gear profile	involute
Input speed of a power source, rpm	1200
Number of life cycles, N	10^8
Design factor of safety, n_d	2,1
Reliability, %	99,9
Operating temperature, T	Moderate or low (~120°C)
Quality number for gear	ANSI/AGMA, 2004 : 9; ISO,2002,2006 :8 Shigley J.E.,1985; Budynas R.G. and Nisbett J.K.,2011; Juvinall R.C. and Marshek K.M.,2011: shaved or ground
Material properties of gear pair	see Table 3.1
Working characteristics of driving and driven machines	Uniform
Selected transmitted power range, kW	1 kW, 10 kW, 100 kW, 500 kW, 1000 kW
Selected Gear speed ratio range, m_G	1:1, 3:1, 5:1, 8:1, 10:1

4.1. The Use of Microsoft Excel Pages

As mentioned in Section 3.3, loss or gain in volume or in selection of material type or stress related performance has been also seen easily on excel pages by changing the parameters. Thus, this provided to determine the loss or the gain between different types of design approaches by obtaining useful charts and/or practical curves using the design results. Figure 4.1 shows an example for the excel page that prepared for spur gear design based on bending fatigue failure by using ANSI/AGMA 2101-D04 Standards (2004).

It consists of input parameters, design variables and the most important design parameters that are module and face width. Design results are directly dependent on the input parameters as mentioned above. And design variables are provided in the form of equations, table and/or figure readings depending on the design approach.

Conventionally, gear box design has always started with the selection of the module, which makes the whole design process iterative, time-consuming and costly. Moreover, the design work requires experience and a great deal of expertise, which is really lacking for novice or inexperienced designer. Hence excel pages were prepared to carry out the design calculations.

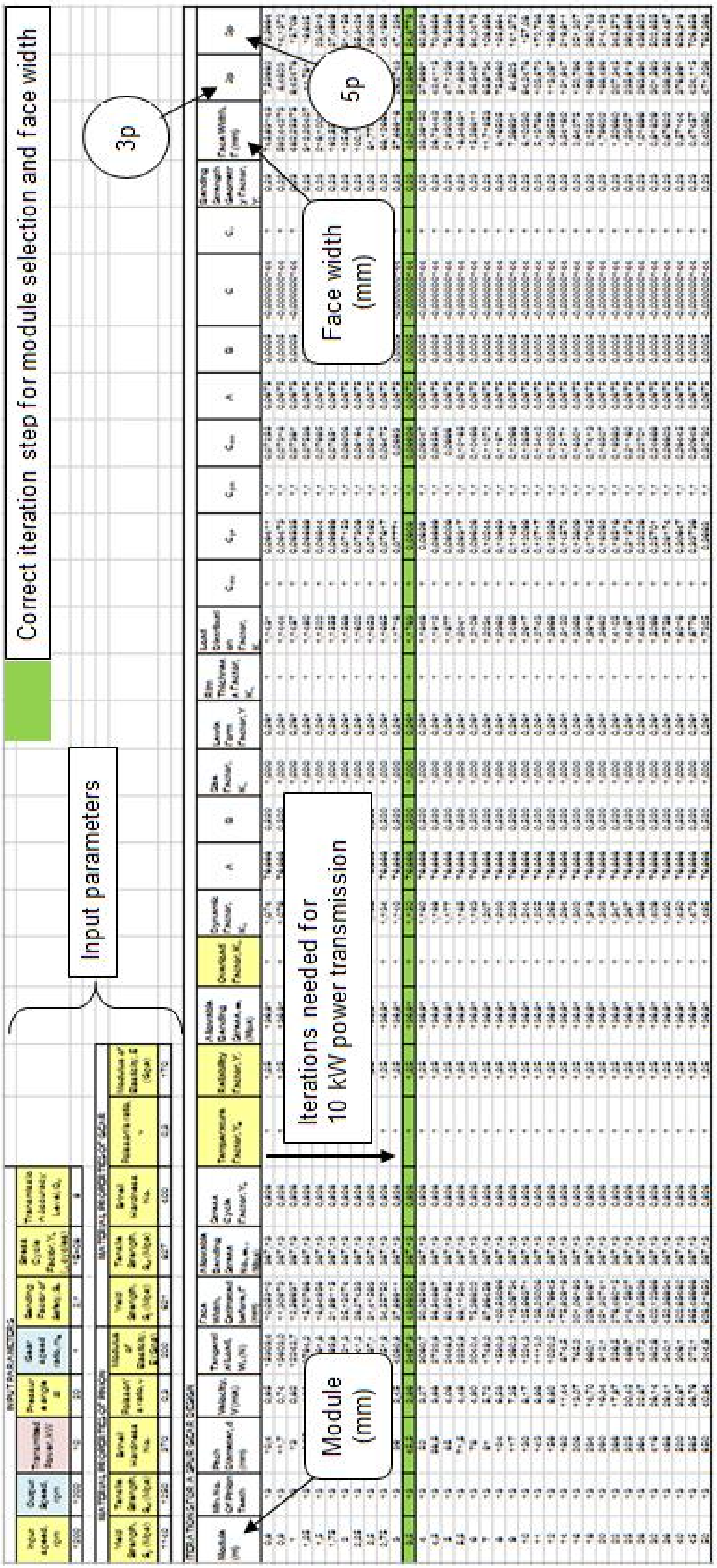


Figure 4.1. A general view of Microsoft Excel page used in this study

As mentioned before, speed reduction by using a spur gear can be achieved up to a gear ratio of 10:1 in a single stage. Hence gear designs are carried out for the gear speed ratio of 1:1, 3:1, 5:1, 8:1 and 10:1. This range may allow plotting results in a curve. Similarly, the power range is selected to cover a wider range. Hence, the designs are carried out for the power transmissions of 1 kW, 10 kW, 100 kW, 500 kW and 1000 kW. Figure 4.2 displays speed ratio and power combinations used in this study. The designs and its results were carried out for the five types of design approaches considering the both bending fatigue and surface contact fatigue separately. This means that for the speed ratio of 1:1, 25 design results are obtained for the bending fatigue and 25 for the surface contact fatigue. This gives total of 5x5x2=50 design results for each of the design approach. The excel pages have provided to obtain these results accurately in short time.

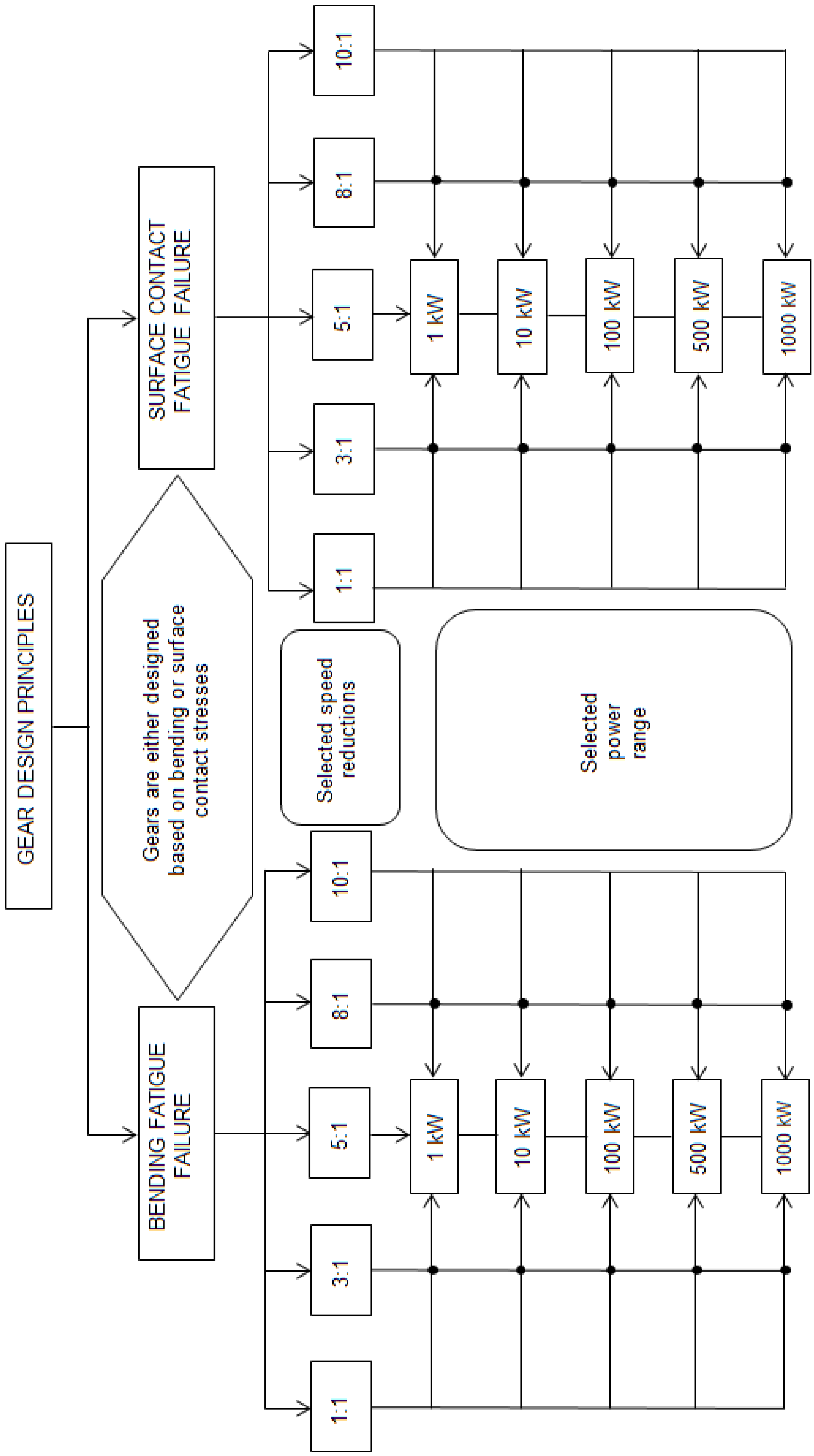


Figure 4.2.Speed ratio and power combinations used in the study for five different types of design approaches

The designer who practicing a spur gear design, does not aware of loss or gain from performance, or material, or volume when using any type of design approach. Therefore following sections provide outputs of the design results. Only the final results of iterations have been given for the representation of findings. Then, the results are used for the comparison of each of the selected gear design approaches.

4.2. Final Iteration Results for Module Selection and Face Width

As represented in Figure 3.2, the iterations for module selection start by estimating a module considering the effect of input parameters. For instance, if the life cycle is desired to be lesser, smaller module can be selected or on the other hand if a moderate gear quality is selected instead of a precision quality one, the module have to be selected bigger. Shigley J.E. (1985) introduced preferable choices for module selections as shown in Table 4.2. By selecting a module from Table 4.2, design variables have been determined for each type of design approaches and iterations are carried out until face width is in between the accepted range (3p ≤F≤ 5p).

Table 4.2. Modules in general use (Shigley J.E., 1985)

MODULES IN GENERAL USE													
Preferred	1	1,25	1,5	2	2,5	3	4	5	6	8	10	12	16
	20	25	32	40	50								
Next Choice	1,125	1,375	1,75	2,25	2,75	3,50	4,50	5,50	7	9	11	14	18
	22	28	36	45									

Table 4.3 to Table 4.7 give the final results of iterations that are obtained for the speed ratio of 1:1, 3:1, 5:1, 8:1 and 10:1, and for the selected power transmission range (1 kW, 10 kW, 100 kW, 500 kW and 1000 kW). In order to see the all results obtained from the five different types of design approaches, compact tables have been prepared. All the results of the designs were found based on both bending stress and surface contact stress fatigue failures using in the selected design approaches. However the minimum number of teeth on pinion that depends on gear speed ratio and pressure angle was selected as given in Tables 4.3 to Table 4.7.

The results are given by comparing the design approaches based on both bending fatigue failure and surface contact fatigue failure criterions. As described in Figure 4.2, comprehensive determinations were carried out to introduce the results in a wider range.

When the tables are closely scrutinized, it is seen that each of the design approach provides close results for module selection and face width. The reason for the variations is mainly due to the effect of design variables and it's mostly due to the inherited features of the each of the design approach that are discussed previously.

Table 4.3. Comparison of module (m) and face width (F) obtained at 1 kW power transmission

SPEED RATIO	NO. of TEETH	DESIGN APPROACHES	module and Face width VALUES for FAILURE CRITERIAS			
			BENDING FATIGUE		SURFACE CONTACT FATIGUE	
			Module (mm)	Face Width (mm)	Module (mm)	Face Width (mm)
1 : 1	13	Shigley J.E., 1985	1,25	20,13683	2,25	24,98269
		Budynas R.G. and Nisbett J.K., 2011	1,25	13,69903	2,5	32,83575
		Juvinall R.C. and Marshek K.M., 2011	1,5	18,70802	3,5	42,76637
		ISO Standards, 2002,2006	1,25	16,08567	3,5	39,54296
		ANSI/AGMA Standards, 2004	1,5	21,91001	3	39,00859
3 : 1	15	Shigley J.E., 1985	1,25	15,52872	1,75	20,57182
		Budynas R.G. and Nisbett J.K., 2011	1	16,93480	2	25,58769
		Juvinall R.C. and Marshek K.M., 2011	1,5	15,00809	2,75	34,49448
		ISO Standards, 2002,2006	1,25	15,10764	2,75	41,73716
		ANSI/AGMA Standards, 2004	1,5	17,36620	2,25	33,74097
5 : 1	16	Shigley J.E., 1985	1,25	13,94572	1,5	22,05702
		Budynas R.G. and Nisbett J.K., 2011	1	15,58389	1,75	26,34735
		Juvinall R.C. and Marshek K.M., 2011	1,25	18,66537	2,5	32,95902
		ISO Standards	1,25	14,64195	2,75	33,79678
		ANSI/AGMA Standards, 2004	1,5	15,0441	2,25	26,65069
8 : 1	17	Shigley J.E., 1985	1,25	12,85342	1,5	18,36851
		Budynas R.G. and Nisbett J.K., 2011	1	14,35301	1,75	21,94491
		Juvinall R.C. and Marshek K.M., 2011	1,25	16,39804	2,5	27,46410
		ISO Standards, 2002,2006	1,25	14,22845	2,5	33,75063
		ANSI/AGMA Standards, 2004	1,25	18,73681	2	27,71840
10 : 1	17	Shigley J.E., 1985	1,25	12,81617	1,5	17,96032
		Budynas R.G. and Nisbett J.K., 2011	1	14,35301	1,75	21,45724
		Juvinall R.C. and Marshek K.M., 2011	1,25	16,39804	2,5	26,85379
		ISO Standards, 2002,2006	1,25	14,23883	2,5	33,26884
		ANSI/AGMA Standards, 2004	1,25	18,73681	2	27,10244

Table 4.4. Comparison of module (m) and face width (F) obtained at 10 kW power transmission

SPEED RATIO	NO. of TEETH	DESIGN APPROACHES	module and Face width VALUES for FAILURE CRITERIAS			
			BENDING FATIGUE		SURFACE CONTACT FATIGUE	
			Module (mm)	Face Width (mm)	Module (mm)	Face Width (mm)
1 : 1	13	Shigley J.E., 1985	3	38,09191	4,5	69,95555
		Budynas R.G. and Nisbett J.K., 2011	2,5	33,42058	5,5	71,11511
		Juvinall R.C. and Marshek K.M., 2011	3,5	35,83486	8	93,26702
		ISO Standards, 2002,2006	2,5	39,42707	7	106,02085
		ANSI/AGMA Standards, 2004	3,5	43,01184	7	79,2458
3 : 1	15	Shigley J.E., 1985	2,75	34,54355	4	41,23518
		Budynas R.G. and Nisbett J.K., 2011	2,25	32,39777	4,5	52,98683
		Juvinall R.C. and Marshek K.M., 2011	3	38,86512	6	82,10994
		ISO Standards, 2002,2006	2,5	36,30481	6	94,19659
		ANSI/AGMA Standards, 2004	3	45,82909	5	74,13589
5 : 1	16	Shigley J.E., 1985	2,5	37,02604	3,5	42,41419
		Budynas R.G. and Nisbett J.K., 2011	2,25	29,84199	4	52,85799
		Juvinall R.C. and Marshek K.M., 2011	3	33,8653	6	65,26852
		ISO Standards	2,5	34,86463	6	76,39045
		ANSI/AGMA Standards, 2004	3	39,74256	5	58,63917
8 : 1	17	Shigley J.E., 1985	2,5	34,15436	3,5	35,36014
		Budynas R.G. and Nisbett J.K., 2011	2,25	27,51026	4	44,07459
		Juvinall R.C. and Marshek K.M., 2011	2,75	35,26066	5,5	64,37549
		ISO Standards, 2002,2006	2,5	33,56922	5,5	75,13715
		ANSI/AGMA Standards, 2004	2,75	41,07738	4,5	59,33126
10 : 1	17	Shigley J.E., 1985	2,5	34,05536	3	46,60429
		Budynas R.G. and Nisbett J.K., 2011	2,25	27,51026	4	43,09516
		Juvinall R.C. and Marshek K.M., 2011	2,75	35,26066	5,5	62,94492
		ISO Standards, 2002,2006	2,5	33,58888	5,5	74,06456
		ANSI/AGMA Standards, 2004	2,75	41,07738	4,5	58,01278

Table 4.5. Comparison of module (m) and face width (F) obtained at 100 kW power transmission

SPEED RATIO	NO. of TEETH	DESIGN APPROACHES	module and Face width VALUES for FAILURE CRITERIAS			
			BENDING FATIGUE		SURFACE CONTACT FATIGUE	
			Module (mm)	Face Width (mm)	Module (mm)	Face Width (mm)
1 : 1	13	Shigley J.E., 1985	7	86,39706	10	150,35833
		Budynas R.G. and Nisbett J.K., 2011	5,5	68,01614	12	158,92284
		Juvinall R.C. and Marshek K.M., 2011	8	72,56826	18	212,36487
		ISO Standards, 2002,2006	5,5	82,45911	16	226,12783
		ANSI/AGMA Standards, 2004	8	91,65051	16	178,69864
3 : 1	15	Shigley J.E., 1985	6	88,78390	8	116,85119
		Budynas R.G. and Nisbett J.K., 2011	5	64,88458	9	139,68559
		Juvinall R.C. and Marshek K.M., 2011	7	81,77848	14	173,88638
		ISO Standards, 2002,2006	5,5	75,89511	14	192,79167
		ANSI/AGMA Standards, 2004	7	93,43790	12	149,24433
5 : 1	16	Shigley J.E., 1985	6	79,91092	8	92,93353
		Budynas R.G. and Nisbett J.K., 2011	5	59,83726	9	111,11457
		Juvinall R.C. and Marshek K.M., 2011	7	71,06721	14	129,20622
		ISO Standards	5,5	72,72111	14	156,89724
		ANSI/AGMA Standards, 2004	7	81,16144	11	138,16036
8 : 1	17	Shigley J.E., 1985	5,5	80,29765	7	100,18759
		Budynas R.G. and Nisbett J.K., 2011	4,5	68,18011	8	116,20663
		Juvinall R.C. and Marshek K.M., 2011	6	84,13396	12	144,69588
		ISO Standards, 2002,2006	5,5	69,78969	12	176,45923
		ANSI/AGMA Standards, 2004	6	94,61758	10	136,40328
10 : 1	17	Shigley J.E., 1985	5,5	80,06490	7	97,96120
		Budynas R.G. and Nisbett J.K., 2011	4,5	68,18011	8	113,62426
		Juvinall R.C. and Marshek K.M., 2011	6	84,13396	12	141,48041
		ISO Standards, 2002,2006	5,5	69,82550	12	173,94026
		ANSI/AGMA Standards, 2004	6	94,61758	10	133,37209

Table 4.6. Comparison of module (m) and face width (F) obtained at 500 kW power transmission

SPEED RATIO	NO. of TEETH	DESIGN APPROACHES	module and Face width VALUES for FAILURE CRITERIAS			
			BENDING FATIGUE		SURFACE CONTACT FATIGUE	
			Module (mm)	Face Width (mm)	Module (mm)	Face Width (mm)
1 : 1	13	Shigley J.E., 1985	12	173,56148	20	215,13721
		Budynas R.G. and Nisbett J.K., 2011	9	125,49902	20	300,68115
		Juvinall R.C. and Marshek K.M., 2011	14	172,18678	32	429,71275
		ISO Standards, 2002,2006	10	130,8449	28	425,49361
		ANSI/AGMA Standards, 2004	14	167,09613	28	344,50467
3 : 1	15	Shigley J.E., 1985	11	136,82062	14	215,09296
		Budynas R.G. and Nisbett J.K., 2011	8	125,37016	16	233,30457
		Juvinall R.C. and Marshek K.M., 2011	12	145,18973	25	348,24699
		ISO Standards, 2002,2006	9	141,68649	25	348,73592
		ANSI/AGMA Standards, 2004	12	175,72313	20	305,84324
5 : 1	16	Shigley J.E., 1985	11	142,24786	14	171,24614
		Budynas R.G. and Nisbett J.K., 2011	8	115,71122	16	185,78781
		Juvinall R.C. and Marshek K.M., 2011	11	149,67324	25	277,56627
		ISO Standards	9	134,60425	25	284,48856
		ANSI/AGMA Standards, 2004	12	152,82958	20	242,72200
8 : 1	17	Shigley J.E., 1985	10	156,23277	14	133,55488
		Budynas R.G. and Nisbett J.K., 2011	8	106,87344	14	200,00112
		Juvinall R.C. and Marshek K.M., 2011	11	131,907	22	295,31493
		ISO Standards, 2002,2006	9	127,95191	22	301,49050
		ANSI/AGMA Standards, 2004	11	156,73352	18	242,19053
10 : 1	17	Shigley J.E., 1985	10	155,77992	14	130,58699
		Budynas R.G. and Nisbett J.K., 2011	8	106,87344	14	195,55665
		Juvinall R.C. and Marshek K.M., 2011	11	131,907	22	288,75238
		ISO Standards, 2002,2006	9	127,99572	22	297,18670
		ANSI/AGMA Standards, 2004	11	156,73352	18	236,80851

Table 4.7. Comparison of module (m) and face width (F) obtained at 1000 kW power transmission

SPEED RATIO	NO. of TEETH	DESIGN APPROACHES	module and Face width VALUES for FAILURE CRITERIAS			
			BENDING FATIGUE		SURFACE CONTACT FATIGUE	
			Module (mm)	Face Width (mm)	Module (mm)	Face Width (mm)
1 : 1	13	Shigley J.E., 1985	16	206,32745	25	338,59791
		Budynas R.G. and Nisbett J.K., 2011	11	167,27475	28	318,42919
		Juvinall R.C. and Marshek K.M., 2011	16	222,56766	40	565,36864
		ISO Standards, 2002,2006	12	181,85262	36	563,45364
		ANSI/AGMA Standards, 2004	18	215,32351	36	457,25116
3 : 1	15	Shigley J.E., 1985	14	190,46640	20	218,80904
		Budynas R.G. and Nisbett J.K., 2011	11	131,78640	20	305,79141
		Juvinall R.C. and Marshek K.M., 2011	16	180,03507	32	437,92805
		ISO Standards, 2002,2006	12	165,17848	32	466,03852
		ANSI/AGMA Standards, 2004	16	211,65594	28	347,81071
5 : 1	16	Shigley J.E., 1985	14	183,96202	18	212,72780
		Budynas R.G. and Nisbett J.K., 2011	10	147,47057	20	243,61874
		Juvinall R.C. and Marshek K.M., 2011	16	146,84201	32	349,21915
		ISO Standards	12	156,58519	32	396,88694
		ANSI/AGMA Standards, 2004	16	184,21942	25	333,30981
8 : 1	17	Shigley J.E., 1985	14	170,16446	16	222,20929
		Budynas R.G. and Nisbett J.K., 2011	10	136,25973	18	248,53301
		Juvinall R.C. and Marshek K.M., 2011	14	178,71137	28	375,34801
		ISO Standards, 2002,2006	11	169,47163	28	400,39668
		ANSI/AGMA Standards, 2004	14	204,41326	22	344,23126
10 : 1	17	Shigley J.E., 1985	14	169,67123	16	217,27130
		Budynas R.G. and Nisbett J.K., 2011	10	136,25973	18	243,01006
		Juvinall R.C. and Marshek K.M., 2011	14	178,71137	28	367,00694
		ISO Standards, 2002,2006	11	169,51295	28	294,65437
		ANSI/AGMA Standards, 2004	14	204,41326	22	336,58168

4.3. Comparison of Module Selection and Face Width Results of the Design Approaches

As described in Figure 4.2, design results were obtained in a wider range (speed ratio from 1:1 to 10:1 and transmitted power from 1 kW to 1000 kW for the design approaches). Thus the results have been compared considering the power transmission ranges and gear speed ratios respectively.

4.3.1. Comparison of Results Considering Power Transmission

At a certain gear speed ratio, if the amount of power transmission is desired to be higher, the module of a pinion has to be selected larger while the material properties are kept identical for each power transmission range. This is because the number of teeth on pinion will be the same at a certain speed ratio but the tangential component of transmitted force will increase.

Curves from Figure 4.3 to Figure 4.12 have been prepared for module selection and face width for the five design approaches. These figures allow to select module and face width for selected speed ratios at any desired power transmission ranges practically for the design approaches.

In this study FEA has been also used to analyze bending stress of 3D spur gears that were modelled with the aid of using the same inputs and using the obtained results of the design approaches. These are discussed in detail in Section 4.4. And considering the Table 4.19 ANSI/AGMA 2101-D04 (2004) design approach that has already been used as a most common standard for the design of a spur gear, give a closer result to FEA results. As a result of this, the following figures are discussed by tables from Table 4.9 to Table 4.18. The ratio of module given by the design approaches to the the module given by ANSI/AGMA (2004) ($m_{design\ approach}/m_{AGMA}$) has obtained and represented in tabular form. The formation of these tables has been explained in Table 4.8.

Table 4.8. The ratio of modules with respect to ANSI/AGMA Standards

Ratio of modules	Abbreviations
$\frac{m_S}{m_{AGMA}}$	m_S : the module obtained by the approach given by Shigley J.E. (1985)
$\frac{m_{B\&N}}{m_{AGMA}}$	$m_{B\&N}$: the module obtained by the approach given by Budynas R.G. and Nisbett J.K. (2011)
$\frac{m_{J\&M}}{m_{AGMA}}$	$m_{J\&M}$: the module obtained by the approach given by Juvinall R.C. and Marshek K.M. (2011)
$\frac{m_{ISO}}{m_{AGMA}}$	m_{ISO} : the module obtained by the approach given by ISO 9085:2002 and 6336 Standards (2002, 2006)
$\frac{m_{AGMA}}{m_{AGMA}}$	m_{AGMA} : the module obtained by the approach given by ANSI/AGMA 2101-D04 Standards (2004)

The differences between modules have been investigated in order to have an idea whether the same behaviors are available or not to mention about a general trends. For this reason, a novel method has been developed as explained in Table 4.8. The method normalizes the modules obtained by the design approaches. In here normalization was made with respect to ANSI/AGMA Standards by dividing the module obtained from the design approaches to the module obtained by ANSI/AGMA Standards, for instance m_S/m_{AGMA} is used.

Practical curves have been represented based on both bending fatigue failure and surface contact fatigue failure respectively and discussions on module have been provided in tabular form.

4.3.1.1. Comparison of Results Based on Bending Fatigue Failure

The following figures and tables provide comparison of results based on bending fatigue failure. Figures have shown the general trends of the design approaches individually. However comparisons are also represented in tabular form but only considering the differences in modules.

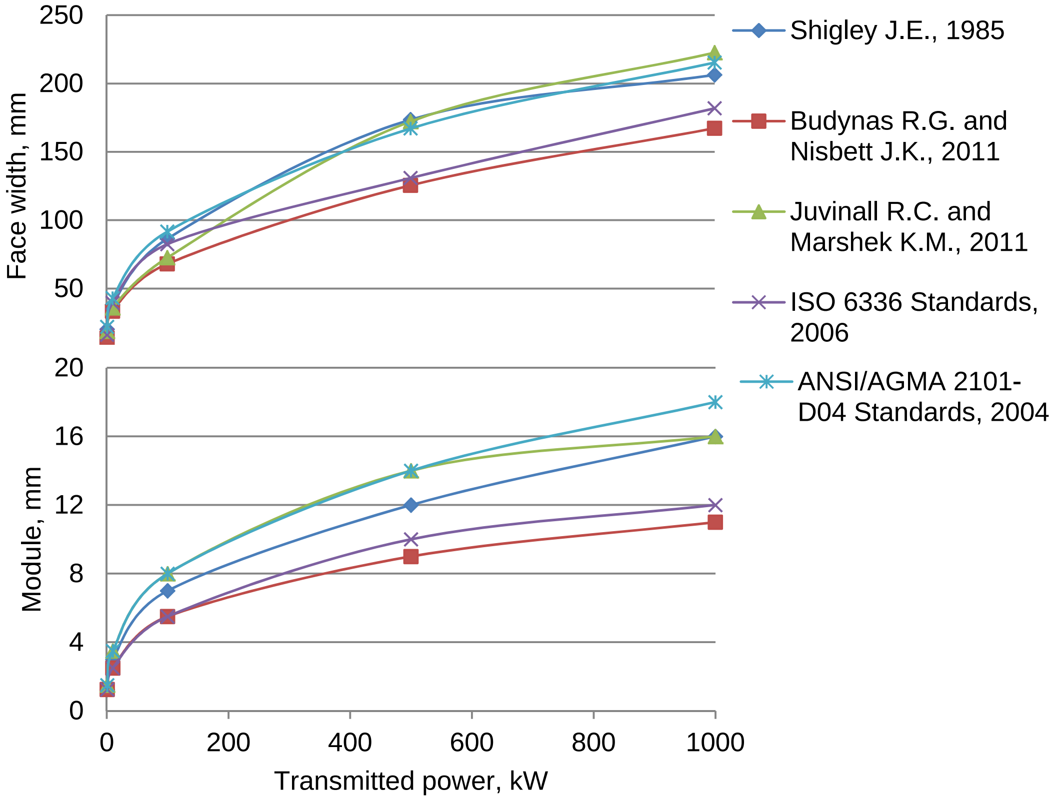


Figure 4.3. Module and face width variation considering bending fatigue failure under increasing power at 1:1 speed ratio

Figure 4.3 shows that the approach given by Budynas and Nisbett (2011) gives the minimum results (module and face width) But in this approach, as mentioned in Section 3.2.2.2, reliability and life for an operation are not being taken into account as well as proving simple determinations.

Table 4.9. The ratio of modules ($m_{design\ approaches}/m_{AGMA}$) based on bending fatigue failure at 1:1 speed ratio

Transmitted power, kW	$\frac{m_S}{m_{AGMA}}$	$\frac{m_{B\&N}}{m_{AGMA}}$	$\frac{m_{J\&M}}{m_{AGMA}}$	$\frac{m_{ISO}}{m_{AGMA}}$	$\frac{m_{AGMA}}{m_{AGMA}}$
1	0,833	0,833	1,000	0,833	1,000
10	0,857	0,714	1,000	0,714	1,000
100	0,875	0,688	1,000	0,688	1,000
500	0,857	0,643	1,000	0,714	1,000
1000	0,889	0,611	0,889	0,667	1,000

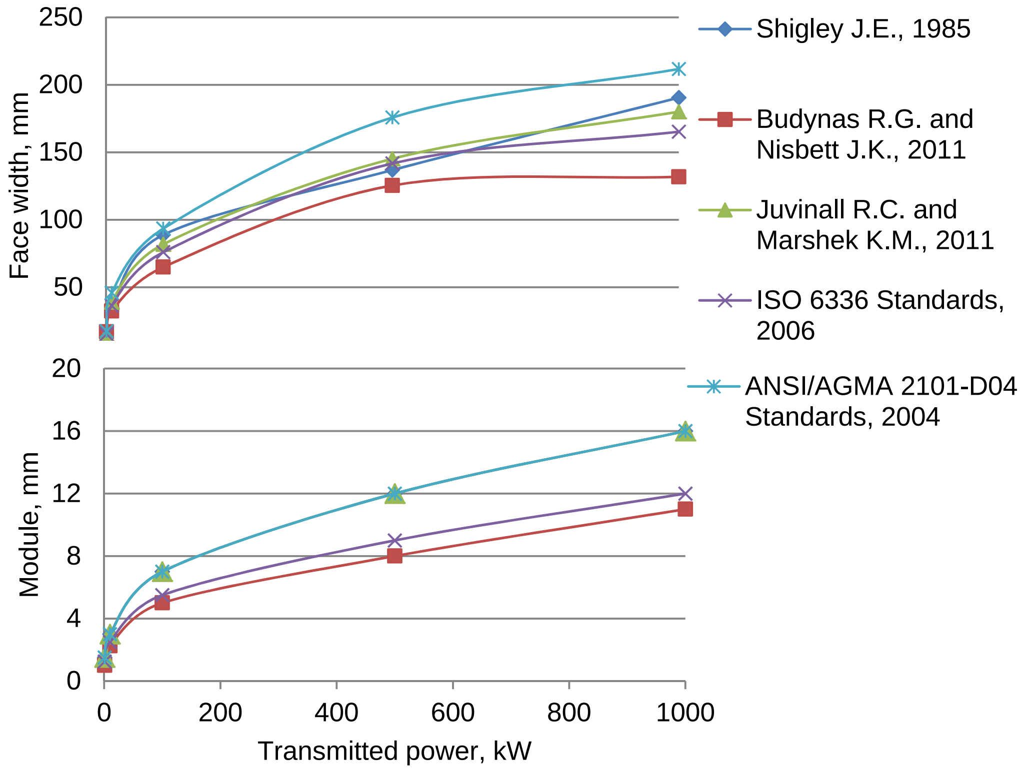


Figure 4.4. Module and face width variation considering bending fatigue failure under increasing power at 3:1 speed ratio

In Figure 4.4 three different approaches given by Shigley J.E. (1985), ANSI/AGMA Standards (2004) and Juvinall R.C. and Marshek K.M. (2011) have given the same modules. Therefore a designer can select one of these approaches which provide an ease of use. But the general trends should be checked for all speed ratios before deciding to use one approach.

Table 4.10. The ratio of modules ($m_{design\ approaches}/m_{AGMA}$) based on bending fatigue failure at 3:1 speed ratio

Transmitted power, kW	$\frac{m_S}{m_{AGMA}}$	$\frac{m_{B\&N}}{m_{AGMA}}$	$\frac{m_{J\&M}}{m_{AGMA}}$	$\frac{m_{ISO}}{m_{AGMA}}$	$\frac{m_{AGMA}}{m_{AGMA}}$
1	0,833	0,667	1,000	0,833	1,000
10	0,917	0,750	1,000	0,833	1,000
100	0,857	0,714	1,000	0,786	1,000
500	0,917	0,667	1,000	0,750	1,000
1000	0,875	0,688	1,000	0,750	1,000

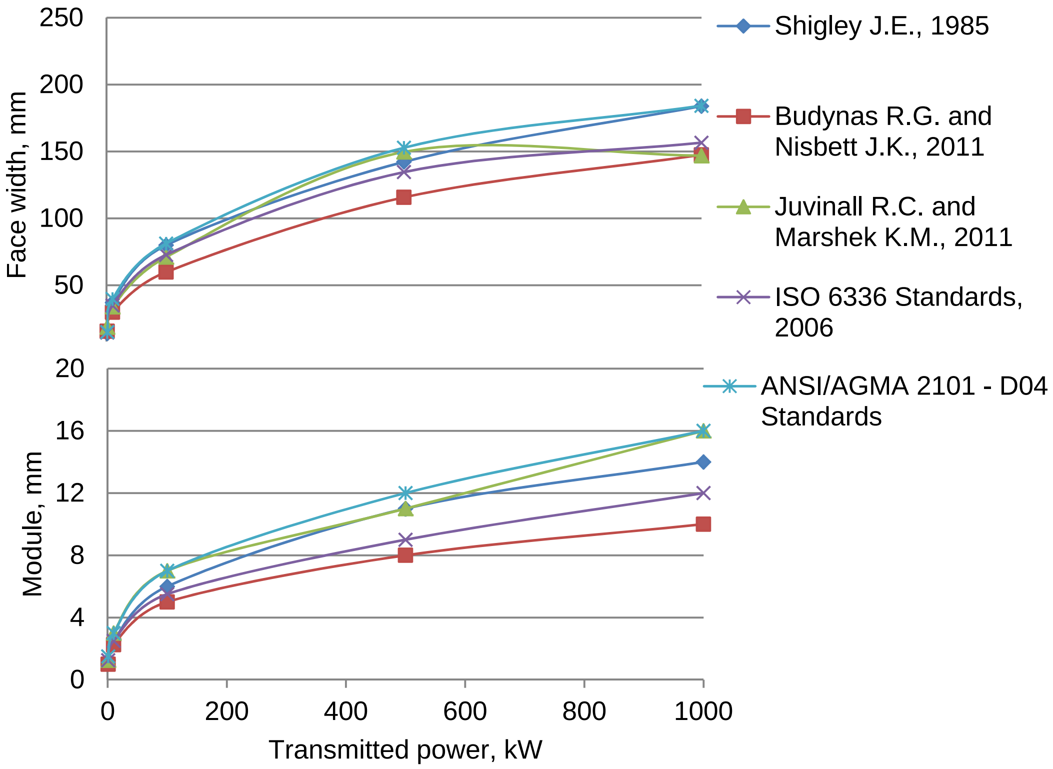


Figure 4.5. Module and face width variation considering bending fatigue failure under increasing power at 5:1 speed ratio

The same trend has still been maintained in Figures 4.5 to 4.7. In these figures, the minimum results have given by Budynas and Nisbett (2011), and by ISO Standards (2006) respectively. However, ISO Standards have better reputation as a standard, hence it may be preferable. Because smaller module means smaller gear size, less material usage, cost effective design and etc.

Table 4.11. The ratio of modules ($m_{design\ approaches}/m_{AGMA}$) based on bending fatigue failure at 5:1 speed ratio

Transmitted power, kW	$\frac{m_S}{m_{AGMA}}$	$\frac{m_{B\&N}}{m_{AGMA}}$	$\frac{m_{J\&M}}{m_{AGMA}}$	$\frac{m_{ISO}}{m_{AGMA}}$	$\frac{m_{AGMA}}{m_{AGMA}}$
1	0,833	0,667	0,833	0,833	1,000
10	0,833	0,750	1,000	0,833	1,000
100	0,857	0,714	1,000	0,786	1,000
500	0,917	0,667	0,917	0,750	1,000
1000	0,875	0,625	1,000	0,750	1,000

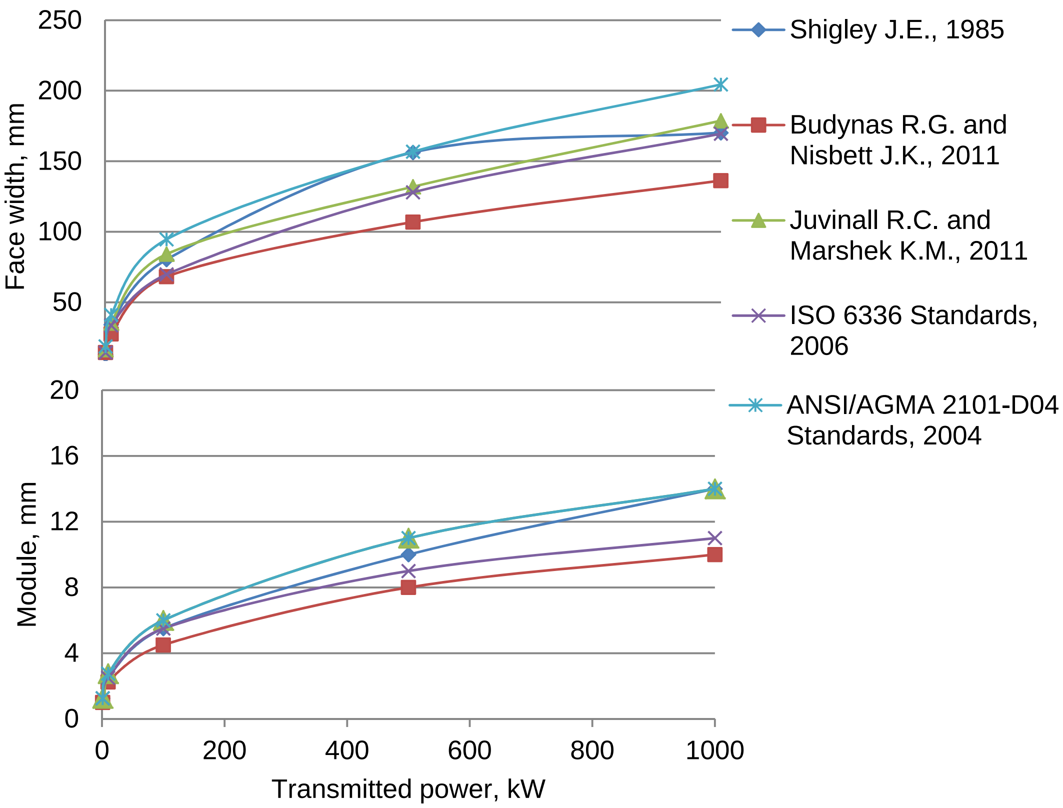


Figure 4.6. Module and face width variation considering bending fatigue failure under increasing power at 8:1 speed ratio

Table 4.12. The ratio of modules ($m_{design\ approaches}/m_{AGMA}$) based on bending fatigue failure at 8:1 speed ratio

Transmitted power, kW	$\frac{m_S}{m_{AGMA}}$	$\frac{m_{B\&N}}{m_{AGMA}}$	$\frac{m_{J\&M}}{m_{AGMA}}$	$\frac{m_{ISO}}{m_{AGMA}}$	$\frac{m_{AGMA}}{m_{AGMA}}$
1	1,000	0,800	1,000	1,000	1,000
10	0,909	0,818	1,000	0,909	1,000
100	0,917	0,750	1,000	0,917	1,000
500	0,909	0,727	1,000	0,818	1,000
1000	1,000	0,714	1,000	0,786	1,000

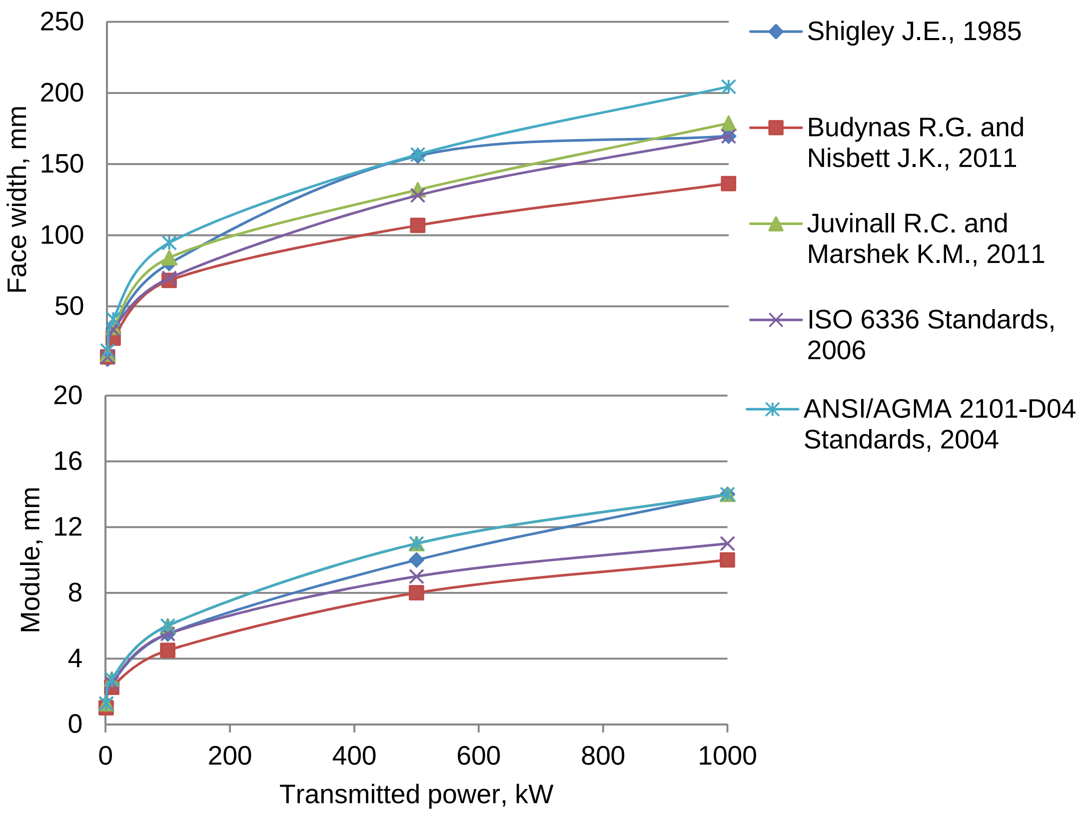


Figure 4.7. Module and face width variation considering bending fatigue failure under increasing power at 10:1 speed ratio

Table 4.13. The ratio of modules ($m_{design\ approaches}/m_{AGMA}$) based on bending fatigue failure at 10:1 speed ratio

Transmitted power, kW	$\frac{m_S}{m_{AGMA}}$	$\frac{m_{B\&N}}{m_{AGMA}}$	$\frac{m_{J\&M}}{m_{AGMA}}$	$\frac{m_{ISO}}{m_{AGMA}}$	$\frac{m_{AGMA}}{m_{AGMA}}$
1	1,000	0,800	1,000	1,000	1,000
10	0,909	0,818	1,000	0,909	1,000
100	0,917	0,750	1,000	0,917	1,000
500	0,909	0,727	1,000	0,818	1,000
1000	1,000	0,714	1,000	0,786	1,000

These figures and tables have shown that it is generally possible to mention about similar trends considering the tables from 4.9 to 4.13. Because when the modules are inversely normalized to ANSI/AGMA Standards, almost the same coefficients have been obtained for the selected speed ratios and power transmission ranges.

Hence, the results are indicated that all the gear design approaches can be corrected to ANSI/AGMA Standards by using a multiplication factor similar to ones as obtained in the above tables. For instance a spur gear can be designed by using Shigley's approach easily and in a short time, and then it is corrected to ANSI/AGMA Standards by an average factor. This provides simplicity for the determinations for novice or beginner users.

However there is another trend that clearly seen between the approach of Juvinall R.C. and Marshek K.M. (2011) and ANSI/AGMA Standards (2004). These two have given almost the same modules although the calculations of ANSI/AGMA Standards are more challenging considering the number of design variables. Also as mentioned in Section 3.2.2.3 approach of Juvinall and Marshek (2011) provides practical curves and/or tables and empirical formulas which provide a simple and quick design for designers.

4.3.1.2. Comparison of Results Based on Surface Contact Fatigue Failure

The following figures and tables provide comparison of results based on surface contact fatigue failure. Comparison of results between the design approaches have been made considering the differences between modules.

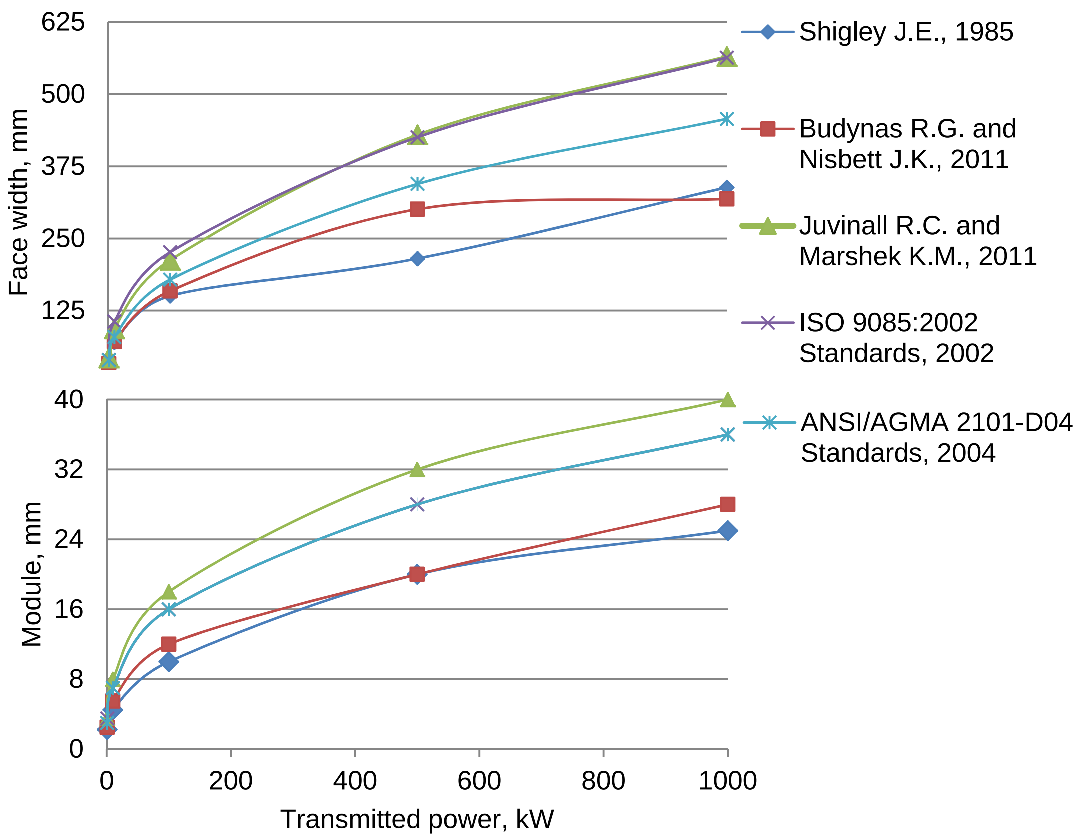


Figure 4.8. Module and face width variation considering surface contact fatigue failure under increasing power at 1:1 speed ratio

Table 4.14. The ratio of modules ($m_{design\ approaches}/m_{AGMA}$) based on surface contact fatigue failure at 1:1 speed ratio

Transmitted power, kW	$\frac{m_S}{m_{AGMA}}$	$\frac{m_{B\&N}}{m_{AGMA}}$	$\frac{m_{J\&M}}{m_{AGMA}}$	$\frac{m_{ISO}}{m_{AGMA}}$	$\frac{m_{AGMA}}{m_{AGMA}}$
1	0,750	0,833	1,167	1,167	1,000
10	0,643	0,786	1,143	1,000	1,000
100	0,625	0,750	1,125	1,000	1,000
500	0,714	0,714	1,143	1,000	1,000
1000	0,694	0,778	1,111	1,000	1,000

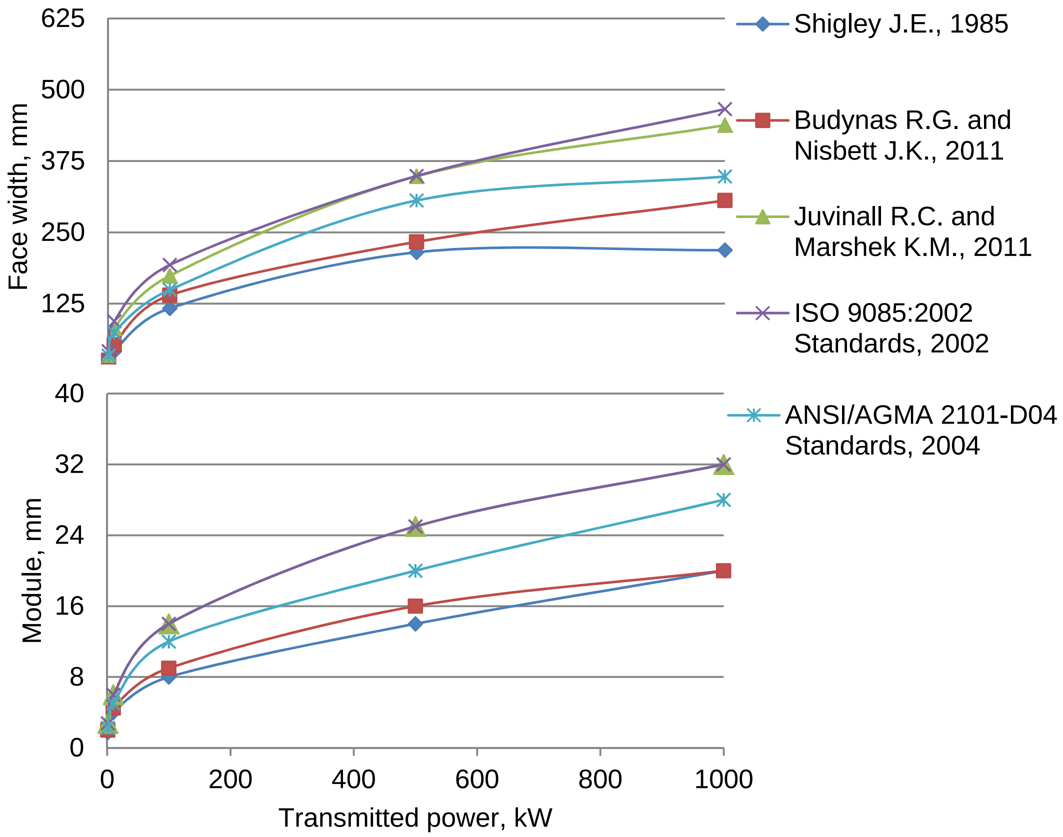


Figure 4.9. Module and face width variation considering surface contact fatigue failure under increasing power at 3:1 speed ratio

For the design criteria of surface contact fatigue failure, ISO 9085:2002 Standards (2002) has given the maximum results except at 1:1 speed ratio. And the minimum results are obtained by Shigley's approach. The reason for this were explained in detail in Section 4.3.2.2.

Table 4.15. The ratio of modules ($m_{design\ approaches}/m_{AGMA}$) based on surface contact fatigue failure at 3:1 speed ratio

Transmitted power, kW	$\frac{m_S}{m_{AGMA}}$	$\frac{m_{B\&N}}{m_{AGMA}}$	$\frac{m_{J\&M}}{m_{AGMA}}$	$\frac{m_{ISO}}{m_{AGMA}}$	$\frac{m_{AGMA}}{m_{AGMA}}$
1	0,778	0,889	1,222	1,222	1,000
10	0,800	0,900	1,200	1,200	1,000
100	0,667	0,750	1,167	1,167	1,000
500	0,700	0,800	1,250	1,250	1,000
1000	0,714	0,714	1,143	1,143	1,000

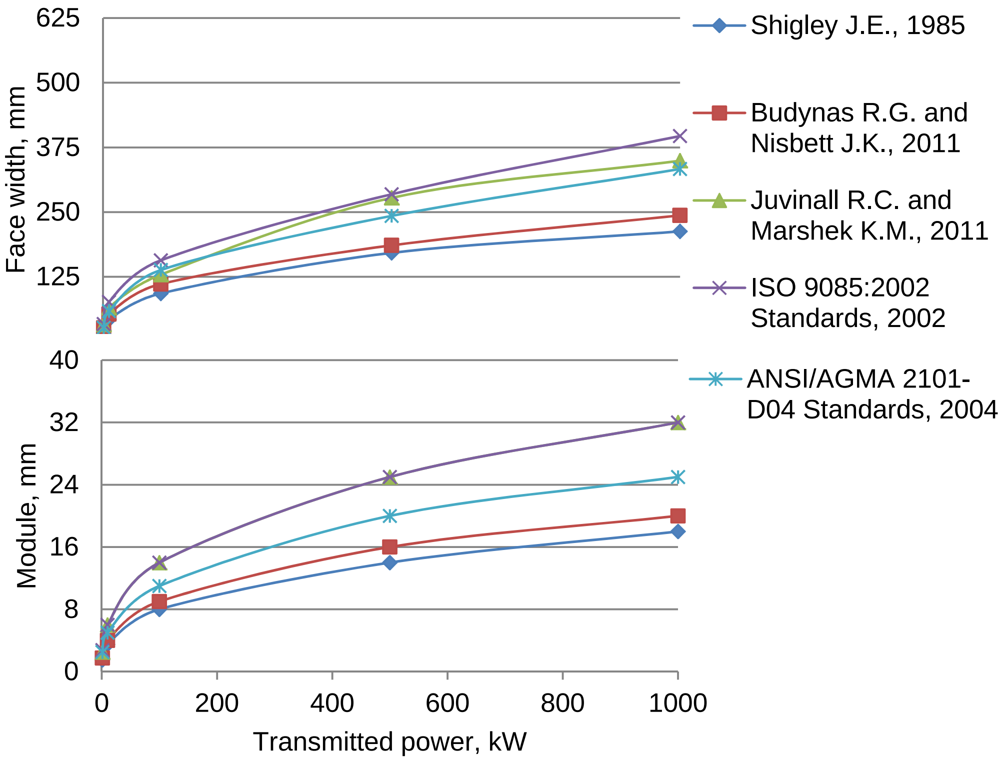


Figure 4.10. Module and face width variation considering surface contact fatigue failure under increasing power at 5:1 speed ratio

It is also possible to mention about a similar trends but in different order as discussed for bending fatigue failure criteria. In this case a designer can select Shigley's approach for a quick design or select ANSI/AGMA Standards to make an optimum design since this standard handles design parameters considering almost all operating conditions and gives results smaller than ISO Standards.

Table 4.16. The ratio of modules ($m_{design\ approaches}/m_{AGMA}$) based on surface contact fatigue failure at 5:1 speed ratio

Transmitted power, kW	$\frac{m_S}{m_{AGMA}}$	$\frac{m_{B\&N}}{m_{AGMA}}$	$\frac{m_{J\&M}}{m_{AGMA}}$	$\frac{m_{ISO}}{m_{AGMA}}$	$\frac{m_{AGMA}}{m_{AGMA}}$
1	0,667	0,778	1,111	1,222	1,000
10	0,700	0,800	1,200	1,200	1,000
100	0,727	0,818	1,273	1,273	1,000
500	0,700	0,800	1,250	1,250	1,000
1000	0,720	0,800	1,280	1,280	1,000

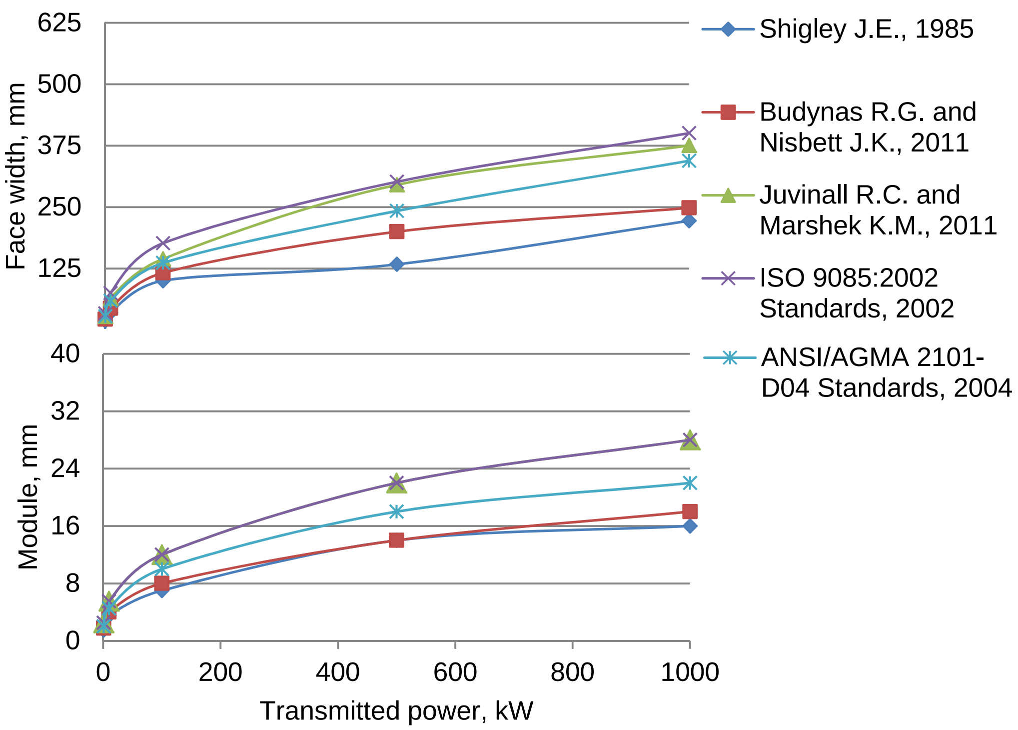


Figure 4.11. Module and face width variation considering surface contact fatigue failure under increasing power at 8:1 speed ratio

When two most common standards (ISO and ANSI/AGMA) are taken into account, ISO Standards has given larger modules for the selected speed ratio and power transmission range except at 1:1 speed ratio. The results are the same for 1:1 speed ratio. The ANSI/AGMA Standards may be preferred when an optimization of gear sizes are desired and provide smaller gear sizes. It should be remembered that ISO Standards are more time-consuming and complicated than ANSI/AGMA as presented in Table 3.29.

Table 4.17. The ratio of modules ($m_{design\ approaches}/m_{AGMA}$) based on surface contact fatigue failure at 8:1 speed ratio

Transmitted power, kW	$\frac{m_S}{m_{AGMA}}$	$\frac{m_{B\&N}}{m_{AGMA}}$	$\frac{m_{J\&M}}{m_{AGMA}}$	$\frac{m_{ISO}}{m_{AGMA}}$	$\frac{m_{AGMA}}{m_{AGMA}}$
1	0,750	0,875	1,250	1,250	1,000
10	0,778	0,889	1,222	1,222	1,000
100	0,700	0,800	1,200	1,200	1,000
500	0,778	0,778	1,222	1,222	1,000
1000	0,727	0,818	1,273	1,273	1,000

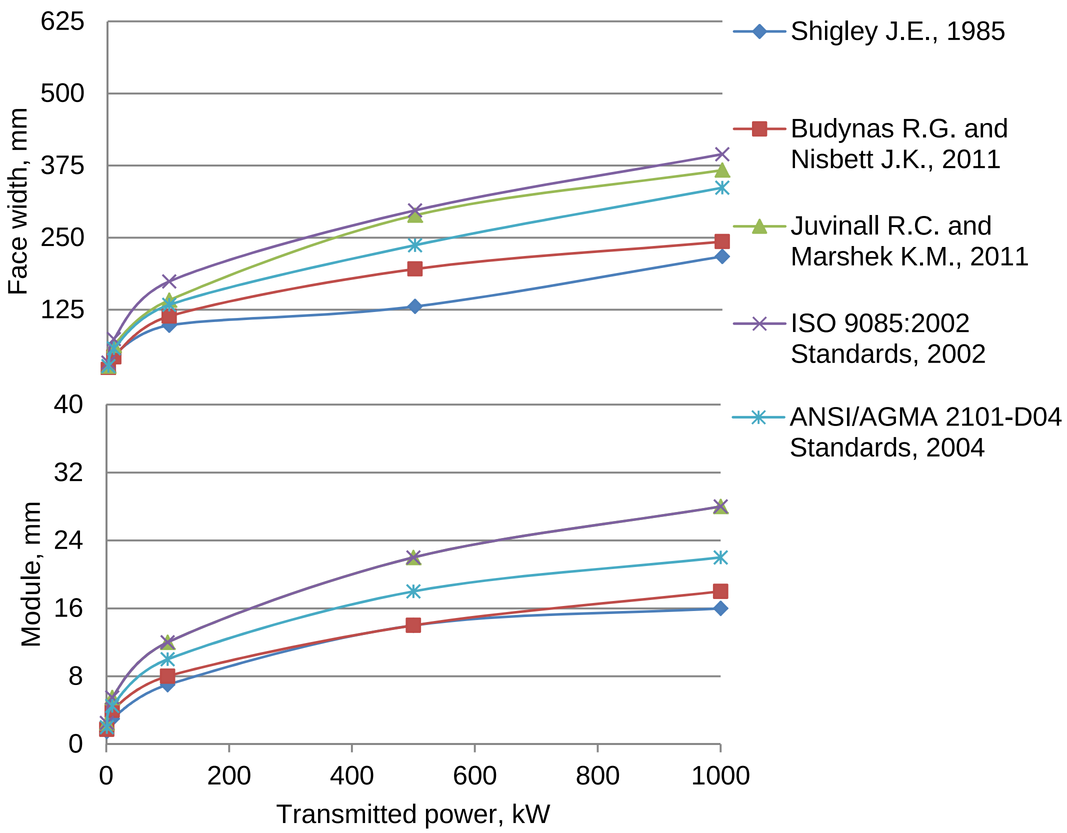


Figure 4.12. Module and face width variation considering surface contact fatigue failure under increasing power at 10:1 speed ratio

Table 4.18. The ratio of modules ($m_{design\ approaches}/m_{AGMA}$) based on surface contact fatigue failure at 10:1 speed ratio

Transmitted power, kW	$\frac{m_S}{m_{AGMA}}$	$\frac{m_{B\&N}}{m_{AGMA}}$	$\frac{m_{J\&M}}{m_{AGMA}}$	$\frac{m_{ISO}}{m_{AGMA}}$	$\frac{m_{AGMA}}{m_{AGMA}}$
1	0,750	0,875	1,250	1,250	1,000
10	0,667	0,889	1,222	1,222	1,000
100	0,700	0,800	1,200	1,200	1,000
500	0,778	0,778	1,222	1,222	1,000
1000	0,727	0,818	1,273	1,273	1,000

On the contrary to the design based on bending fatigue failure, the design approach given by Juvinall R.C. and Marshek K.M. (2011) has given the same module with ISO 9085:2002 Standards (2002) at almost all selected ranges while designing a spur gear based on surface contact fatigue failure.

When the curves are generally examined, nearly the same results are obtained at power transmission ratios of 1 kW and 10 kW. And it seen that general trend is similar for all range of gear speed ratios. However, the design approaches provided different trends over the transmitted power of 100 kW. This is because of the varying design variables and inherited features of the each of design approaches. However, there is still an opportunity to correct results to ANSI/AGMA 2101-D04 or ISO 9085:2002 (2002) Standards by using the multiplication factor similar to ones as obtained in the above tables.

Now, multiplication factors presented in tables from Table 4.9 to Table 4.18 were collected for each of the approach at a certain speed ratio considering the failures criteria. The result have been represented independent of the amount of power transmission for each of the approach respectively.

Table 4.19. Multiplication factors for Shigley J.E.'s approach

speed ratio	BENDING FATIGUE		SURFACE CONTACT FATIGUE	
	average multiplication factor (μ)	standard deviation (σ)	average multiplication factor (μ)	standard deviation (σ)
1:1	0,862	0,019	0,685	0,046
3:1	0,880	0,033	0,732	0,050
5:1	0,863	0,031	0,703	0,021
8:1	0,947	0,043	0,747	0,030
10:1	0,947	0,043	0,724	0,039

Table 4.20. Multiplication factors for Budynas R.G. and Nisbett J.K.'s approach

speed ratio	BENDING FATIGUE		SURFACE CONTACT FATIGUE	
	average multiplication factor (μ)	standard deviation (σ)	average multiplication factor (μ)	standard deviation (σ)
1:1	0,698	0,077	0,772	0,040
3:1	0,697	0,032	0,811	0,074
5:1	0,685	0,043	0,799	0,013
8:1	0,762	0,041	0,832	0,043
10:1	0,762	0,041	0,832	0,043

Table 4.21. Multiplication factors for Juvinall R.C. and Marshek K.M.'s approach

speed ratio	BENDING FATIGUE		SURFACE CONTACT FATIGUE	
	average multiplication factor (m)	standard deviation (s)	average multiplication factor (m)	standard deviation (s)
1:1	0,978	0,044	1,138	0,019
3:1	1,000	0,000	1,196	0,038
5:1	0,950	0,067	1,223	0,063
8:1	1,000	0,000	1,233	0,025
10:1	1,000	0,000	1,233	0,025

Table 4.22. Multiplication factors for the approach of ISO Standards

speed ratio	BENDING FATIGUE		SURFACE CONTACT FATIGUE	
	average multiplication factor (m)	standard deviation (s)	average multiplication factor (m)	standard deviation (s)
1:1	0,723	0,058	1,033	0,067
3:1	0,790	0,037	1,196	0,038
5:1	0,790	0,037	1,245	0,030
8:1	0,886	0,076	1,233	0,025
10:1	0,886	0,076	1,233	0,025

4.3.2. Comparison of Results Considering Speed Ratio

Speed ratio affects the number of teeth on a gear while meshing with a pinion. As it is represented by tables in Section 4.2, selection of proper module for an involute spur gear decreases if the gear speed ratio is desired to be higher. This is because the number of teeth on a gear increases which is in a mesh while running with a pinion. And the force exerted on each tooth on a pinion decreases. Thus gear stresses decreases and the module can be selected smaller for a better design.

The effect of speed ratio on the selection of module has varied for the design approaches too. At a certain amount of power transmission, comparison of module selections is given on bar charts. The charts of Figure 4.13 to 4.17 were created for bending fatigue failure criterion. The charts of Figure 4.18 to 4.22 were created for surface contact fatigue failure criteria.

4.3.2.1. Comparison of Results Based on Bending Fatigue Failure

When spur gears are designed based on bending fatigue failure it is seen that the differences between the design approaches are larger at a speed ratio of 1:1 for the selected power transmission ranges. If the speed ratio increases these differences are getting smaller, and the results given by the design approaches are getting closer to each other as represented in figures below.

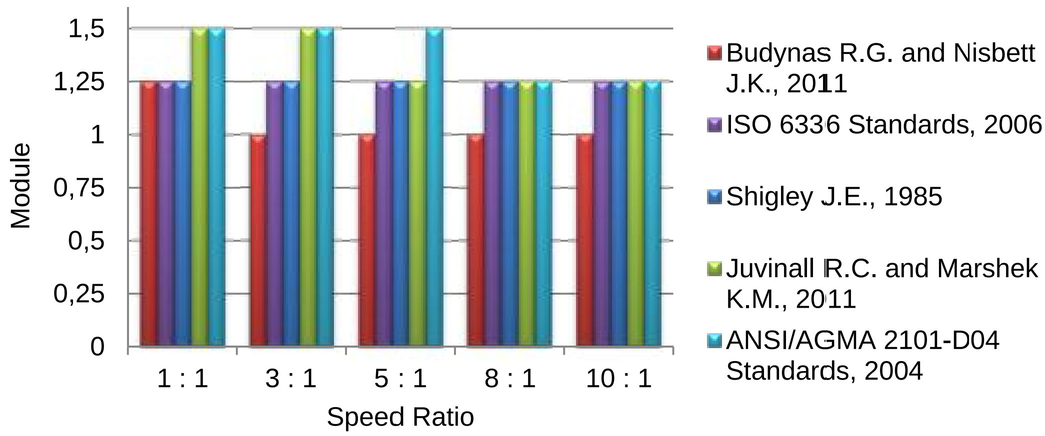


Figure 4.13. The effect of speed ratio on module selection based on bending fatigue failure at 1 kW power transmission

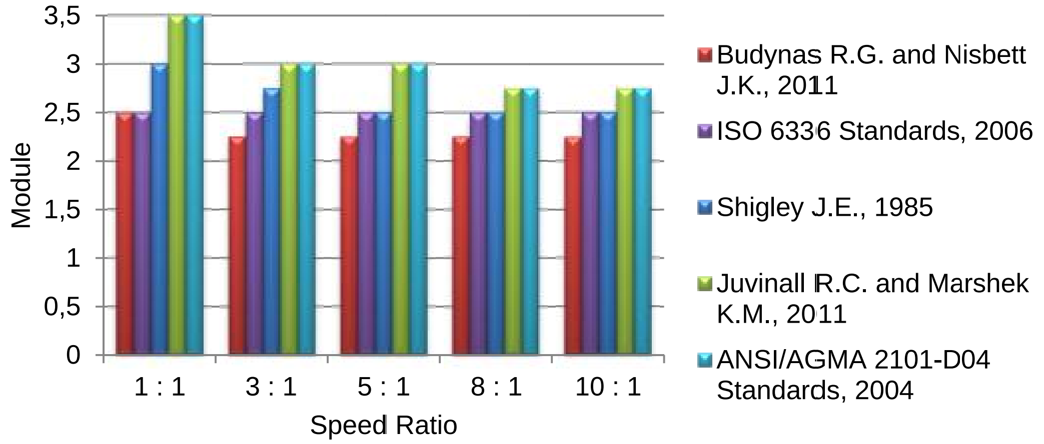


Figure 4.14. The effect of speed ratio on module selection based on bending fatigue failure at 10 kW power transmission

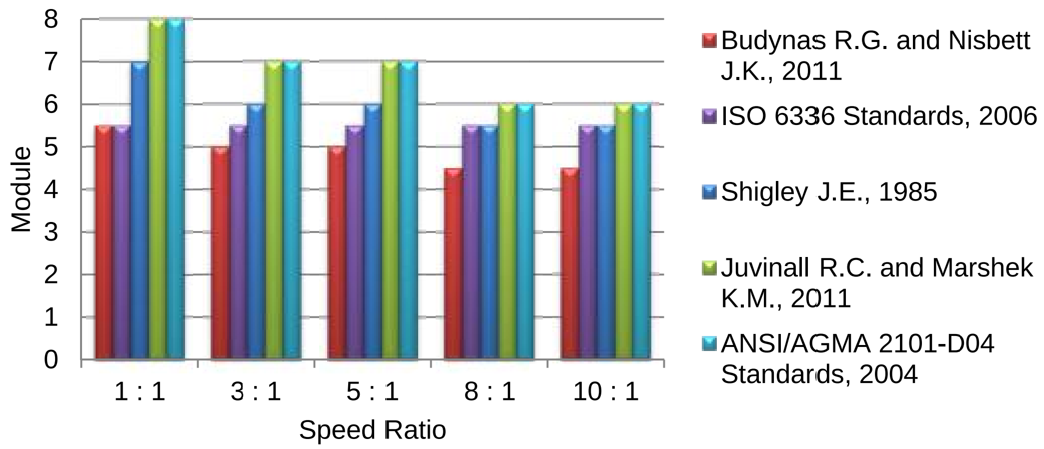


Figure 4.15. The effect of speed ratio on module selection based on bending fatigue failure at 100 kW power transmission

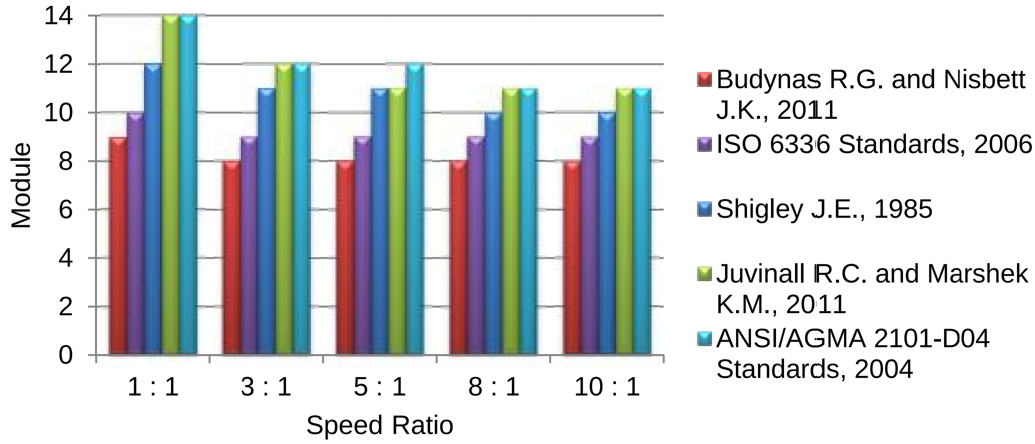


Figure 4.16. The effect of speed ratio on module selection based on bending fatigue failure at 500 kW power transmission

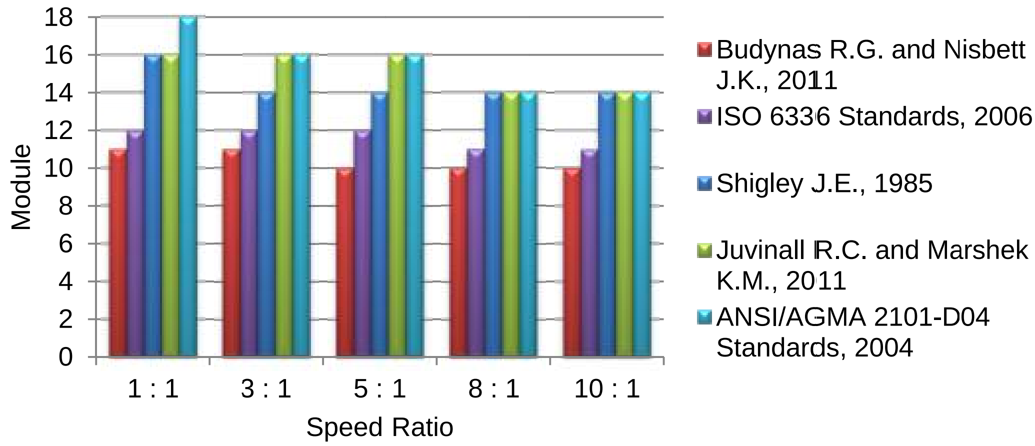


Figure 4.17. The effect of speed ratio on module selection based on bending fatigue failure at 1000 kW power transmission

When the above diagrams are scrutinized, the general trend is that modules decrease for almost all power transmission ranges as the speed ratio increases. However the reduction of module for ISO Standards takes place above 100 kW. This means that the effect of speed ratio on the module selection by using ISO Standards is not seen up to a power transmission of 100 kW. All the design approaches except ISO Standards have shown the similar trends. But for the power transmission of 500 kW and 1000 kW, ISO standard has been in a good agreement with the others.

4.3.2.2. Comparison of Results Based on Surface Contact Fatigue Failure

When designing a pinion based on surface contact fatigue, the design factor of safety is applied to tangential force by its square except the design approach given by Shigley J.E., (1985). As a result of this, Shigley's (1985) gear design approach gives the lowest module selection when compared to other type of design approaches as shown in figures below. However in this case the difference between the design approaches have become larger.

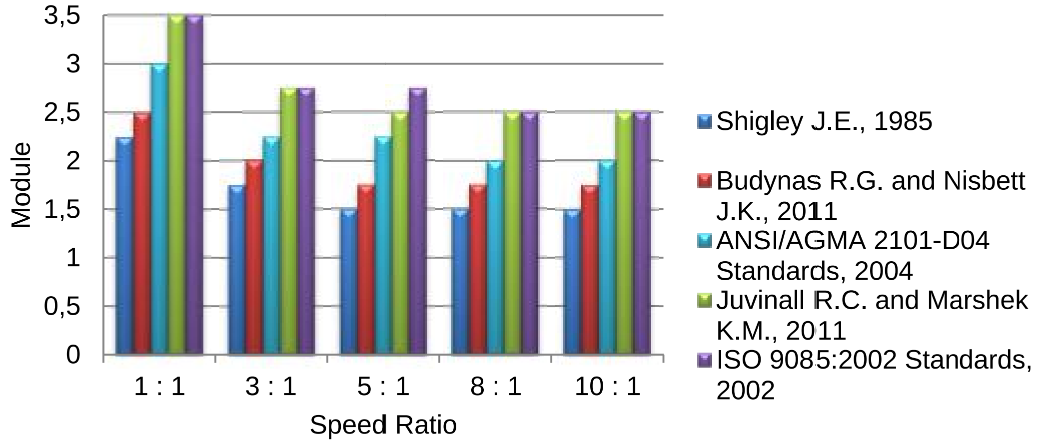


Figure 4.18. The effect of speed ratio on module selection based on surface contact fatigue failure at 1 kW power transmission

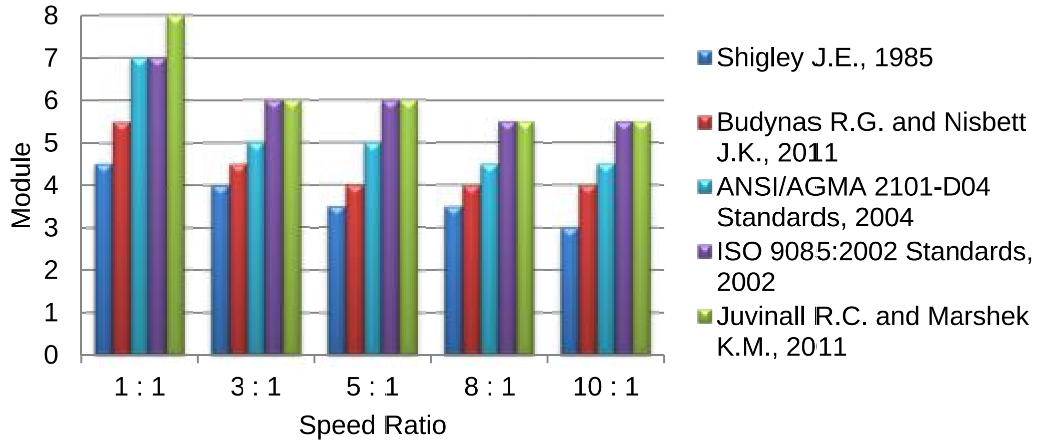


Figure 4.19. The effect of speed ratio on module selection based on surface contact fatigue failure at 10 kW power transmission

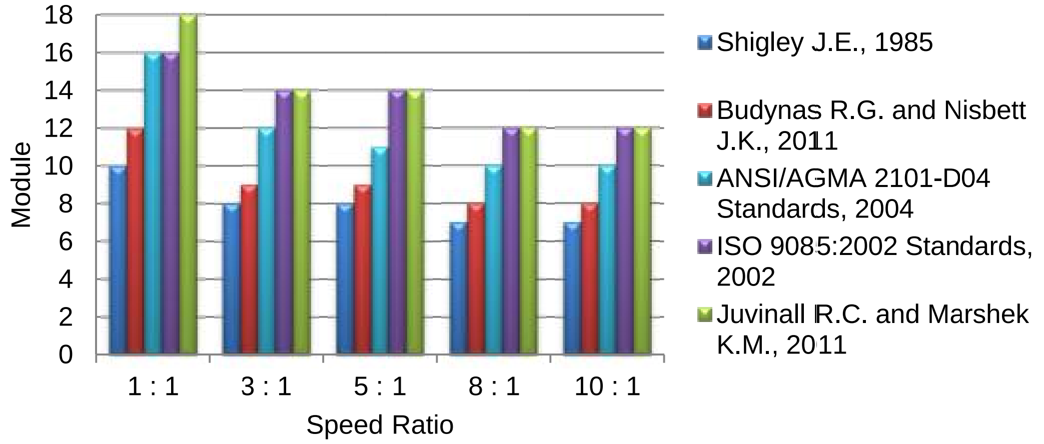


Figure 4.20. The effect of speed ratio on module selection based on surface contact fatigue failure at 100 kW power transmission

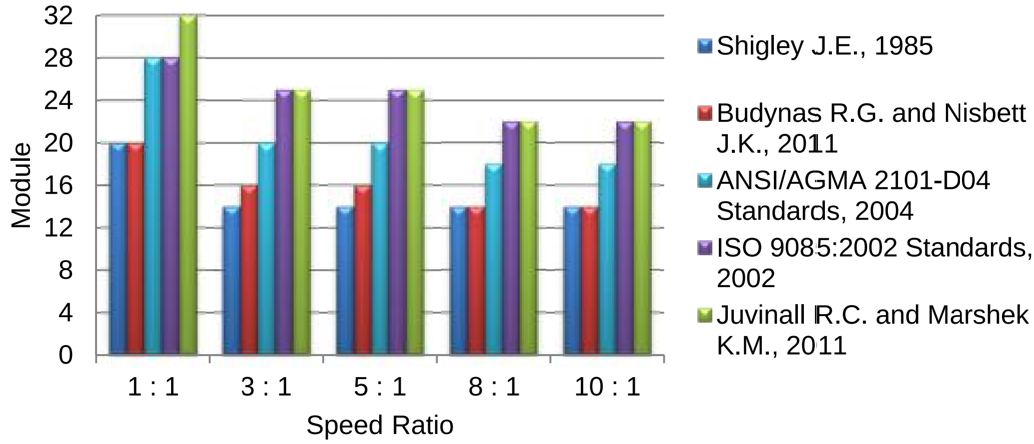


Figure 4.21. The effect of speed ratio on module selection based on surface contact fatigue failure at 500 kW power transmission

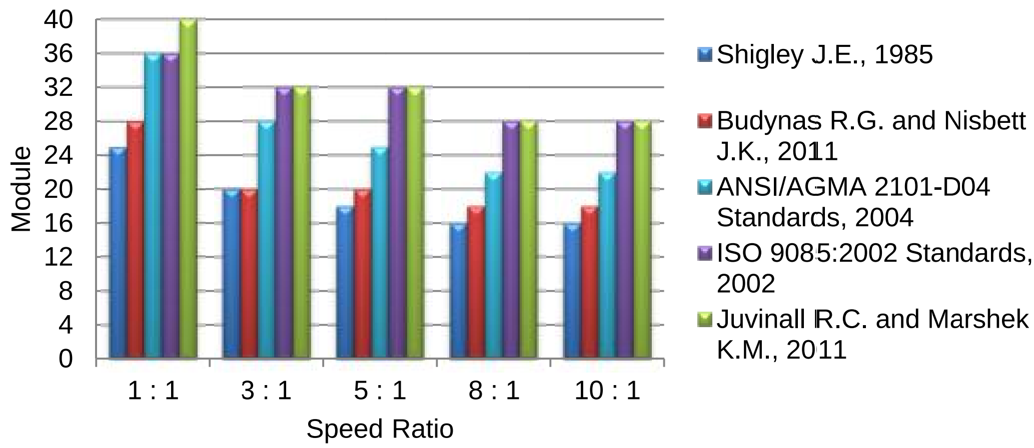


Figure 4.22. The effect of speed ratio on module selection based on surface contact fatigue failure at 1000 kW power transmission

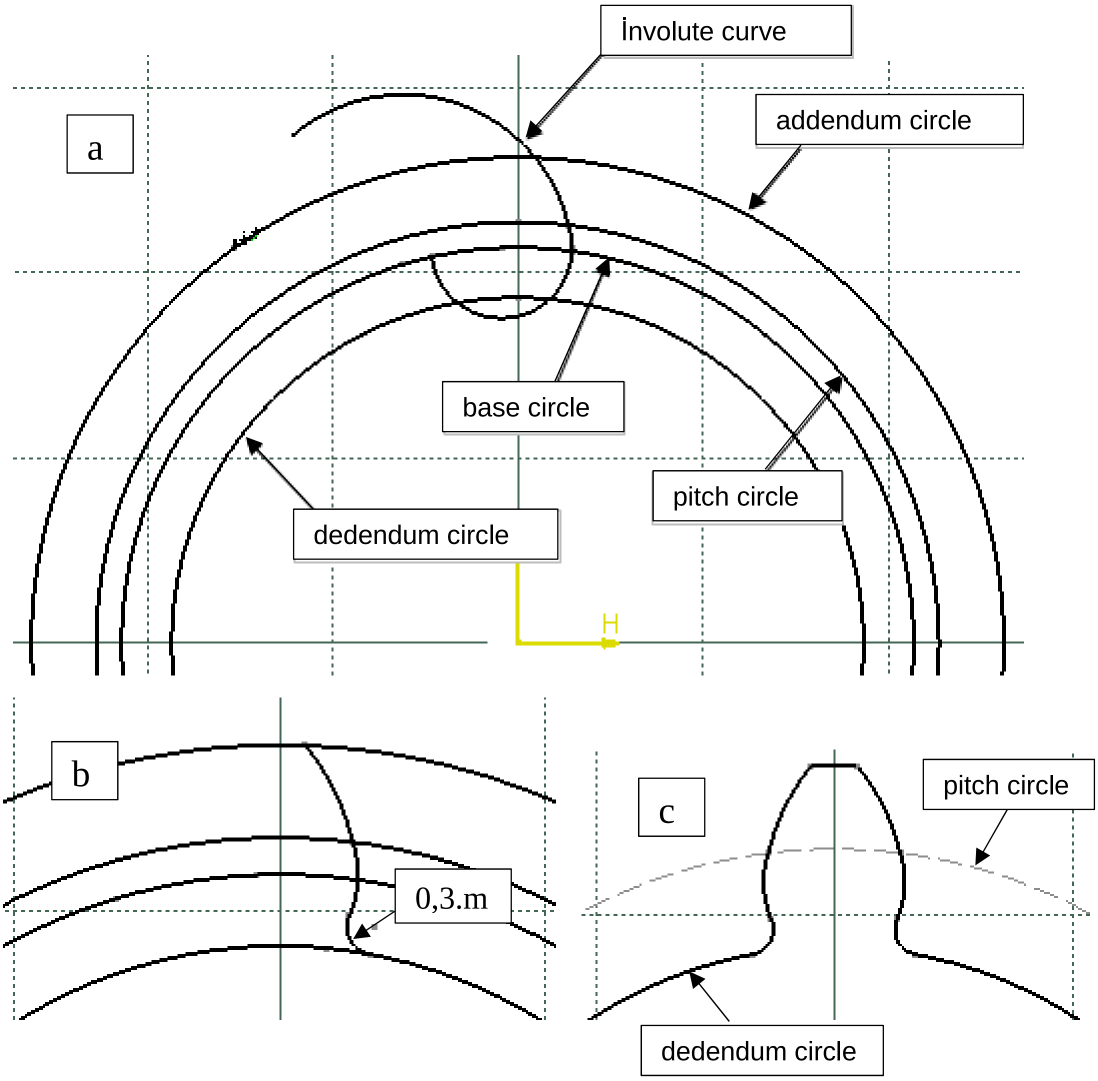
When designing the gear based on surface contact fatigue failure, it is not possible to mention about a general trend for the design approaches as in bending fatigue failure. In here, the trends between design approaches are different and the results are not converging between each other. However it is seen from bar charts that there is no change between the speed ratios of 8:1 and 10:1 at all selected power transmission ranges for all the design approaches. This is because the number of meshing teeth or the contact ratio for the speed ratio of 8:1 and 10:1 are almost the same. Another trend is also was found to exist for all the speed ratios in all the selected power ranges.

The trend is that all the gear design approaches were found to be in the same pattern (except for the power of 1 kW). The gear design approaches were ranked for the increasing modules as in a pattern of;

1) Shigley J.E., 1985
2) Budynas R.G. and Nisbett J.K., 2011
3) ANSI/AGMA 2101-D04 Standards, 2004
4) ISO 9085:2002 Standards, 2002
5) Juvinall R.C. and Marshek K.M., 2011

4.4. Comparison of Gear Stress by Using a Finite Element Method (FEM)

A general approach for using the FEM was mentioned in Section 3.4. In here the creation of geometrical form of a gear and the setup of analysis problem has been shown in figures step by step. Figure 4.23 shows the creation of a gear by using an involute curve; Figure (a), represents the involute curve of a spur gear that is created considering the module, number of teeth and pressure angle, (b) and (c) are the formation of gear tooth profile, (d) is the 2D model of an involute spur gear and (e) is the 3D model of an involute spur gear that is analyzed in ANSYS Workbench 14.0.


a
İnvolute curve
addendum circle
base circle
pitch circle
dedendum circle
H
b
0,3.m
c
pitch circle
dedendum circle

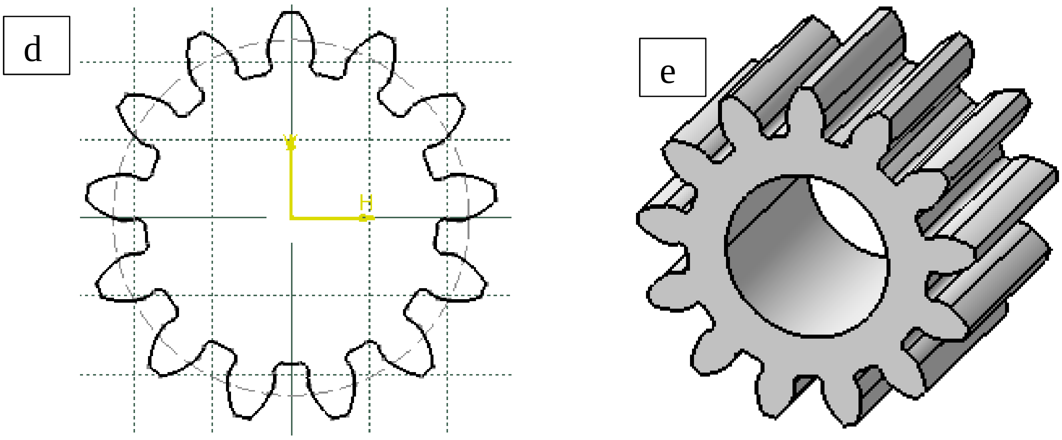


Figure 4.23. Generating an involute spur gear by using the design parameters

After the geometry was created it was imported to ANSYS as a file format of “*Standard for the Exchange of Product”* (STP). Then material properties that mentioned in Table 3.1 were defined in the window of engineering data in Workbench 14.0. Boundary conditions for the structure were defined as pinion that is subjected to tangential load to the pitch diameter along its axis with frictionless support as seen in Figure 4.24. In this figure, (a) shows the mesh elements, which is the subject of preprocessing in ANSYS. Figure 4.24 (b) shows the boundary conditions, and the tangential load is applied along the pitch line as represented in (c). And Figure 4.24 (d) shows the post processing in which the results are obtained.

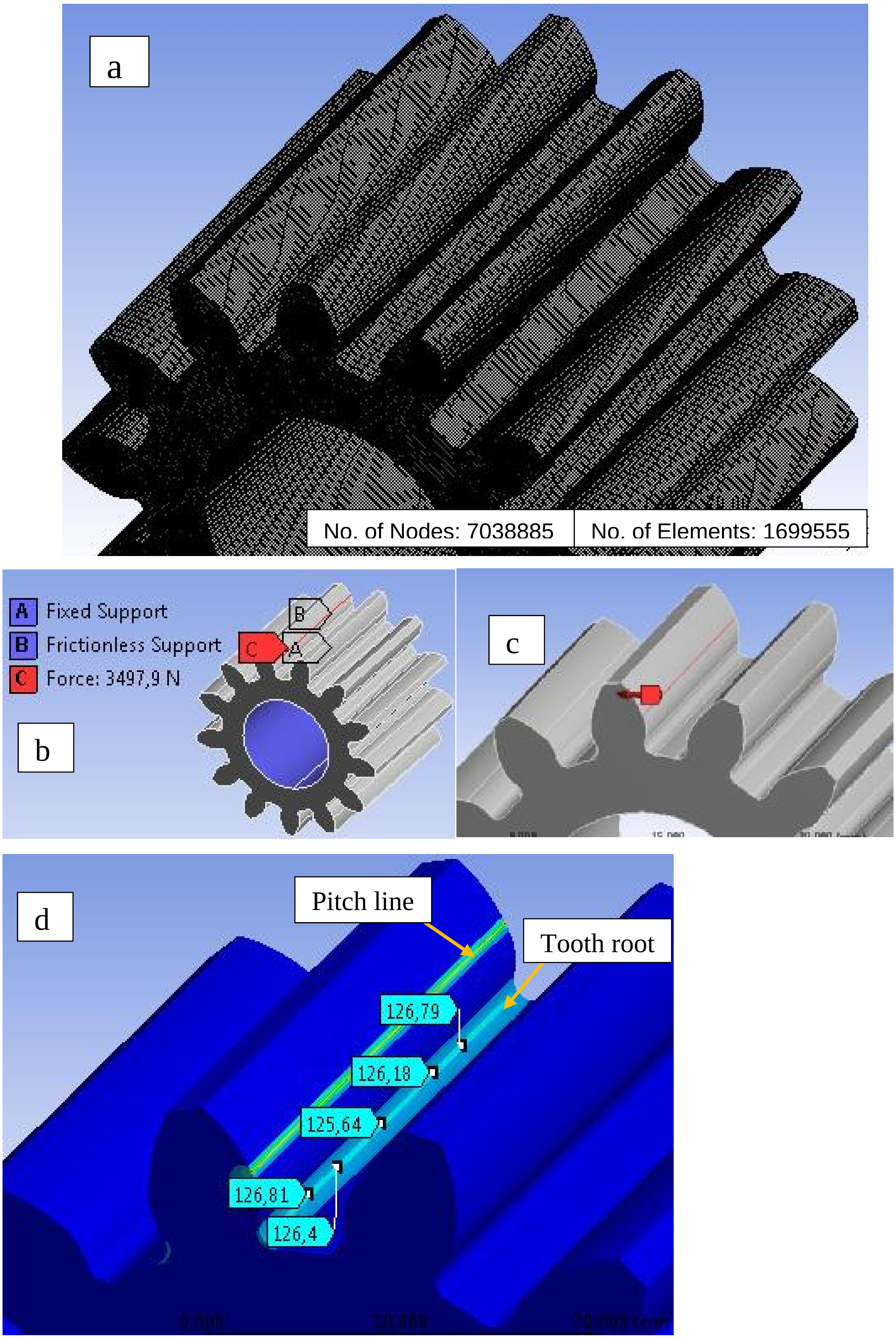


Figure 4.24. Preprocessing, solver and post processing steps in ANSYS Workbench 14.0

By using the final design results, m and F, gear bending stress has been determined numerically on the software of ANSYS Workbench 14.0. Spur gears were modelled for the design approaches using the same design input parameters. And the results of finite element analysis (FEA) have been given comparatively with analytical results in Table 4.23.

Table 4.23. Comparison of bending stresses obtained from the five analytical approaches and a numerical (FEA) method

Design Approaches	Module (mm)	Results for Bending Stress		Difference (%)
		Analytical (MPa)	Numerical (MPa)	
Shigley J.E., 1985	3	142,4	187,9	31,95
Budynas R.G. and Nisbett J.K., 2011	2,5	251,74	253,23	0,59
Juvinall R.C. and Marshek K.M., 2011	3,5	180,02	156,64	12,9
ISO 6336 Standards, 2006	2,5	248,73	268,79	8
ANSI/AGMA 2101-D04 Standards, 2004	3,5	136,89	126,66	7,36

Table 4.23 has shown that the design approach given by Budynas R.G. and Nisbett J.K. (2011) has given the closest result but in this approach the reliability factor, life cycle factor for operation and the load distribution factor are not taken into account as recommended by the approach itself while running excel iterations. For this reason ANSI/AGMA Standards has been taken as a base solution for making comparisons between design approaches since the design variables are handled broadly and gives a closer result to FEA.

4.5. Comparison of Combined Module and Face width for Design Approaches

In this section, the combination of module and face width is given together. Because both module and face width have to be taken into consideration for making a geometrical optimization. Therefore it is going to be very useful to see the total effect of both face width and module on the gear design results. For this, m´ F results are combined to obtain a more like a geometrical value.

The results (m, F) of ANSI/AGMA 2101-D04 Standards have been taken as a reference since the most reliable solution is obtained by FEA. And the following ratio has been defined to compare the results of design approaches;

$$\frac{m_i \times F_i}{m_0 \times F_0}$$

Where m_i and F_i are the module and face width obtained for the target gear design approach respectively, and where m_0 and F_0 are the module and face width obtained from ANSI/AGMA 2101-D04 Standards respectively. Hence, the geometrical results have been normalized with respect to ANSI/AGMA Standards.

The ratios rate the results to indicate the most or worst design. Obviously, the smaller ratios will be the good indicator of better or cost effective design approaches. These were made below for fatigue failures criteria.

4.5.1. Comparison of m´ F over m_0´ F_0 ratios for Bending Fatigue Failure

In this section the design results of m times F (m´ F), were obtained based on bending fatigue failure. The values of m and F were multiplied and divided to the product of results (m_0´ F_0) obtained from ANSI/AGMA Standards. Comparison of m_i´ F_i/m_0´ F_0 values that are obtained by using the design approaches are presented by preparing radar charts. The charts are prepared and presented for the selected range of speed ratio for the selected power range.

The results presented in the charts allow to compare the each of the gear design approach for the overall size.

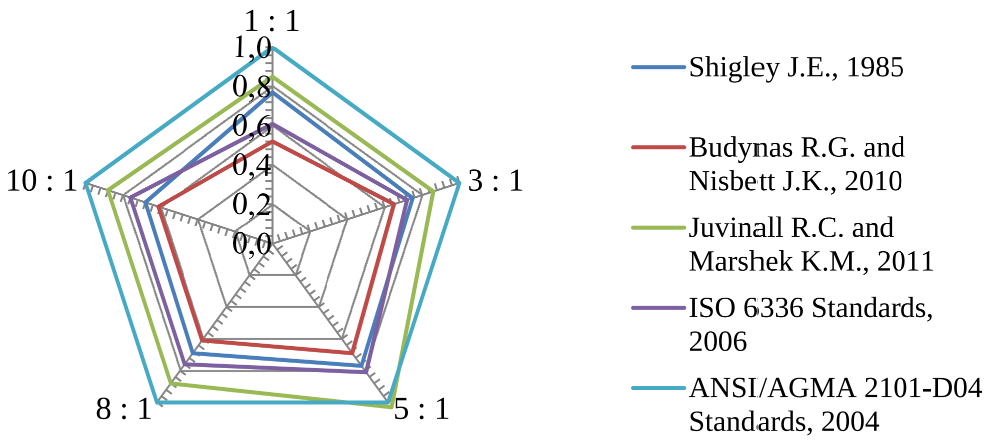


Figure 4.25. Comparison of $m' F/m_0' F_0$ ratios for the design approaches at 1 kW power transmission

Since the ANSI/AGMA Standard gives the highest results it is placed to the outermost in Figure 4.25 except for 5:1 speed ratio. And similar trends have been shown in the following figures.

In order to examine this kind of charts it is better to separate the speed ratios by zones. The following table has defined the zones.

Table 4.24. Definition of zones by speed ratios

Range of speed ratios	**Definition of zones**
1:1 – 3:1	Zone I
3:1 – 5:1	Zone II
5:1 – 8:1	Zone III
8:1 – 10:1	Zone IV

As it is seen from Table 4.24, the region between 10:1 and 1:1 has not been defined. This is because it is truly representative. Therefore charts should be examined with the order of zone I-II-III-IV.

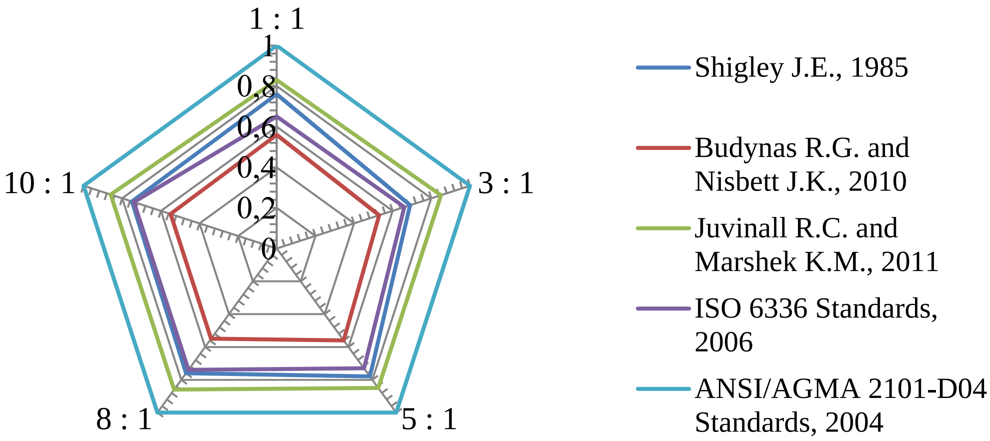


Figure 4.26. Comparison of $m' F/m_0' F_0$ ratios for the design approaches at 10 kW power transmission

When the Figures from 4.25 to 4.29 are observed, the results from the smallest to the bigger is ranked as;

1) Budynas R.G. and Nisbett J.K., 2011
2) ISO 6336 Standards, 2006
3) Shigley J.E., 1985
4) Juvinall R.C. and Marshek K.M., 2011
5) ANSI/AGMA 2101-D04 Standards, 2004

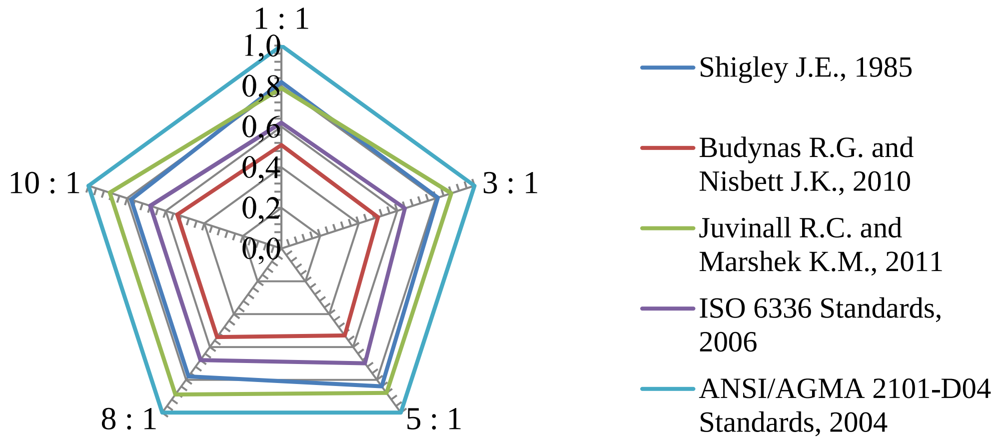


Figure 4.27. Comparison of m´ F/m$_0$´ F$_0$ ratios for the design approaches at 100 kW power transmission

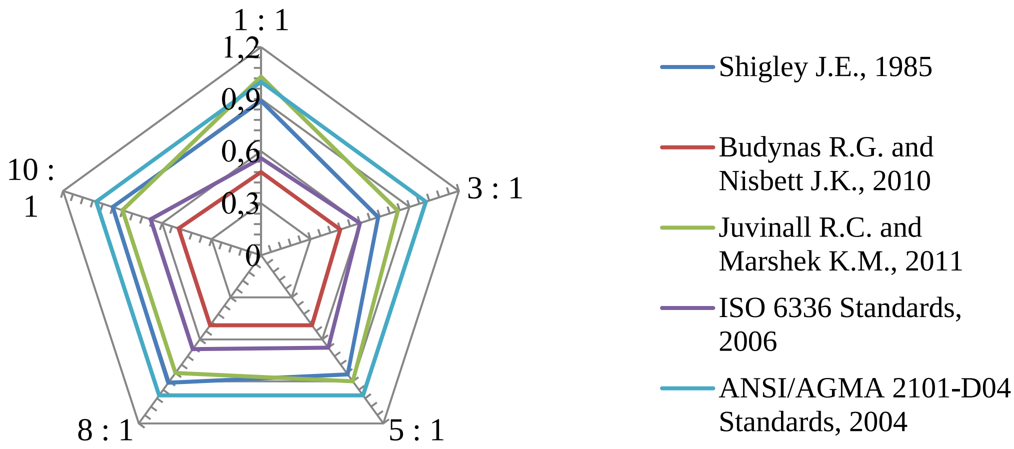


Figure 4.28. Comparison of m´ F/m$_0$´ F$_0$ ratios for the design approaches at 500 kW power transmission

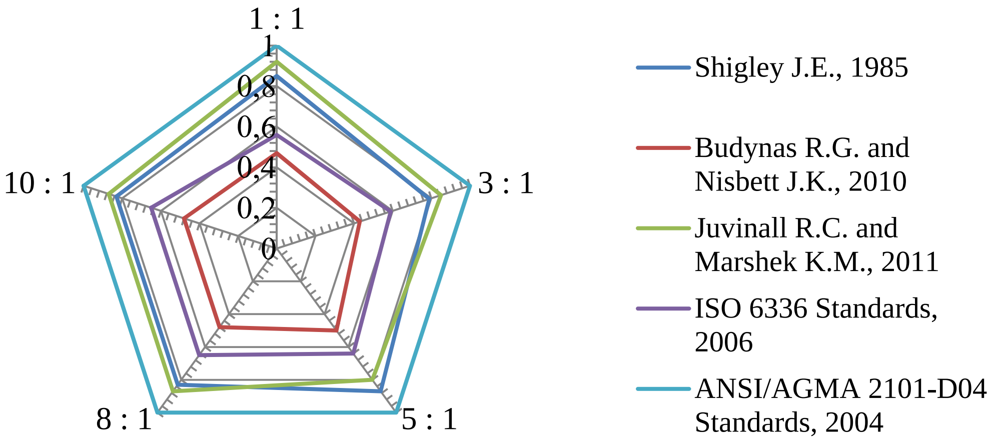


Figure 4.29. Comparison of $m' F/m_0' F_0$ ratios for the design approaches at 1000 kW power transmission

When the figures are examined, it is seen that the design of spur gear by using ANSI/AGMA Standards (2004) gives the largest size for the selected speed ratios for the selected power transmission ranges. And the approach given by Budynas R.G. and Nisbett J.K. (2011) gives the smallest size for a spur gear considering the same conditions with ANSI/AGMA. However these figures have indicated that the results of ISO Standards which are also commonly used such as ANSI/AGMA Standards, are smaller than ANSI/AGMA that means smaller in size. This infers that better geometrical optimization can be achieved by using ISO Standards when the design is carried out based on bending fatigue failure.

4.5.2. Comparison of $m' F$ over $m_0' F_0$ ratios for Surface Contact Fatigue Failure

In this section the design results of m times F ($m' F$), were obtained based on surface contact fatigue failure. The values of m and F were multiplied and divided to the product of results ($m_0' F_0$) obtained from ANSI/AGMA Standards. And the results of gear design approaches have been represented comparatively in Figures 4.30 to 4.34.

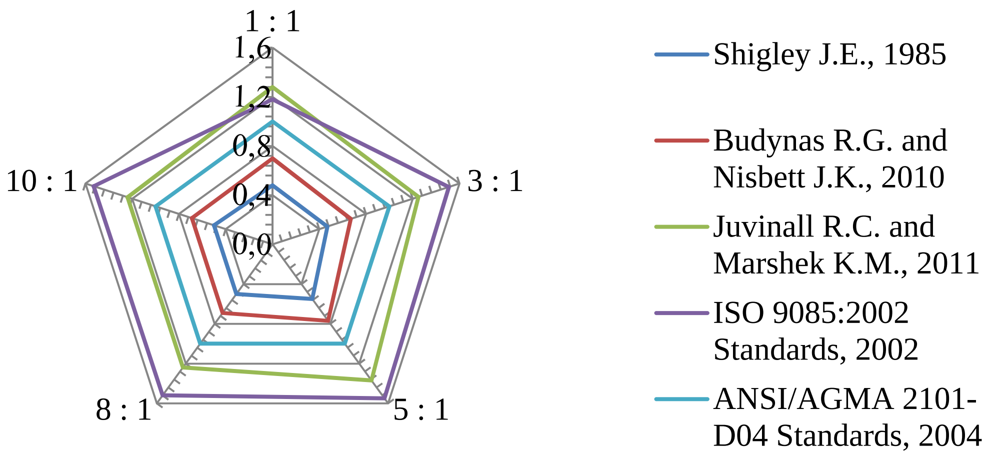


Figure 4.30. Comparison of m´ F/m$_0$´ F$_0$ ratios for the design approaches at 1 kW power transmission

When the figures from 4.30 to 4.34 are examined, except 1:1 speed ratio for all power transmission ranges and 10:1 speed ratio at 1000 kW, the results from the smallest to the bigger is ranked as;

1) Shigley J.E., 1985
2) Budynas R.G. and Nisbett J.K., 2011
3) ANSI/AGMA 2101-D04 Standards, 2004
4) Juvinall R.C. and Marshek K.M., 2011
5) ISO 9085:2002 Standards, 2002

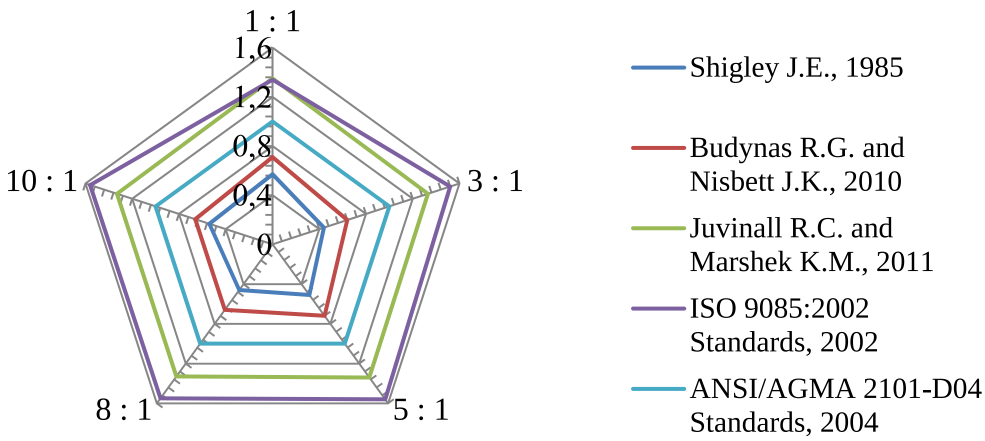


Figure 4.31. Comparison of m´ F/m_0´ F_0 ratios for the design approaches at 10 kW power transmission

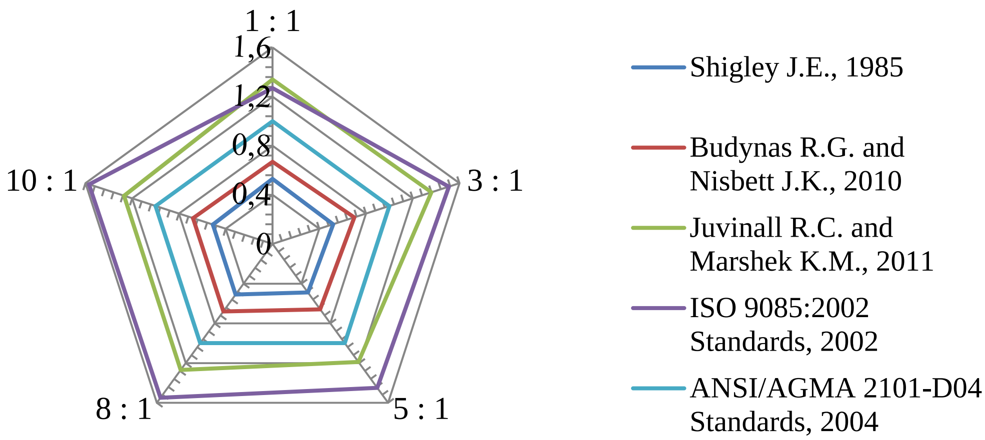


Figure 4.32. Comparison of m´ F/m_0´ F_0 ratios for the design approaches at 100 kW power transmission

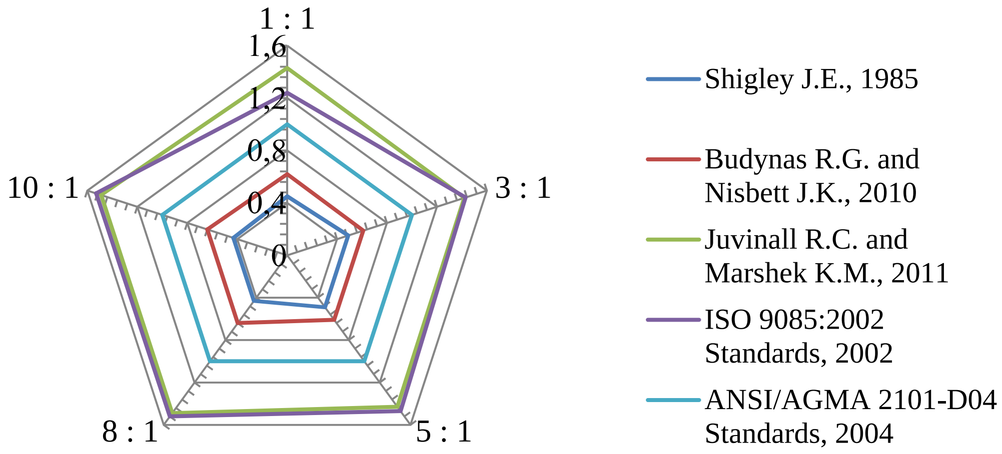


Figure 4.33. Comparison of m´ F/m$_0$´ F$_0$ ratios for the design approaches at 500 kW power transmission

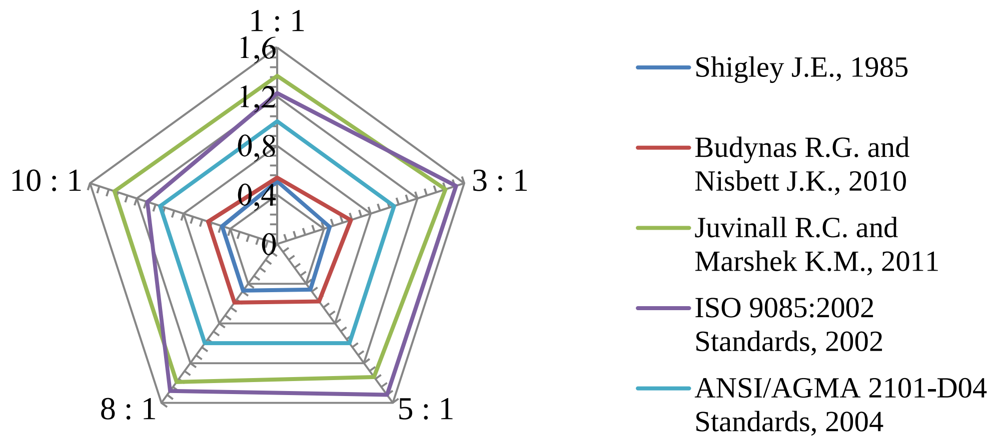


Figure 4.34. Comparison of m´ F/m$_0$´ F$_0$ ratios for the design approaches at 1000 kW power transmission

Conversely to the bending fatigue failure, ISO 9085:2002 Standards (2002) and the approach given by Juvinall R.C. and Marshek K.M. (2011) have given the largest size for a spur gear that is designed based on surface contact fatigue failure. And the approach given by Shigley J.E. (1985) has given the smallest size, explained in Section 4.3.2.2. However as discussed in Section 4.3.2.2, Juvinall and Marshek's gear design approach has given the same modules as in ISO Standards but when the figures are observed from Figure 4.30 to Figure 4.34 the effect of face width is clearly seen. The difference of face width values affects the overall size differently even the modules are the same as presented by figures in Section 4.3.1.2.

Therefore all these radar charts except for the speed ratio of 1:1 have also shown that design approaches have maintain a general trends even the effect of face width has included. But for the design approach of Juvinall and Marshek (2011) at a power transmission of 1000 kW the influence of face width is distinctly larger than in ISO Standards for 1:1 and at 10:1 speed ratios even the modules are found to be equal to each other as in Figures 4.9 to 4.12.

However these charts have indicated the design approaches that provide a spur gear with the minimum size. This allows to designer to estimate the amount of material as well as the overall cost of gear box. And if the optimum material selection using proper material selection approaches or tools such as one given by Ashby M.F. (2010) can be made for a pair of gear, the optimization of a gear box design can be achieved completely.

5. CONCLUSION

This thesis meets a need of selecting and using appropriate involute spur gear design approaches for all designers including the expert designers and novice learners who are practicing a spur gear design. This was made by comparing the most commonly used involute spur gear design approaches available in the literature. The selected approaches are given as follow;

1- Mechanical Engineering Design 1st Metric Edition, Shigley J.E., 1985
2- Shigley's Mechanical Engineering Design 9th Edition (SI), Budynas R.G. and Nisbett J.K., 2011
3- Fundamental of Machine Component Design 5th Edition, Juvinall R.C. and Marshek K.M., 2011
4- ISO 6336 Standards, 2006 and ISO 9085:2002 Standards, 2002
5- ANSI/AGMA 2101-D04 Standards, 2004

Computational load of the approaches considering the number of relevant pages and design variables with its sub variables are presented in Table 5.1.

Table 5.1. Computational load of the approaches

DESIGN APPROACHES	Number of relevant pages	Number of Design Variables+Sub Design Variables	
		bending fatigue failure	surface contact fatigue failure
Mechanical Engineering Design 1st Metric Ed.	13	11+0	11+0
Shigley's Mechanical Engineering Design 9th Ed.	11	9+6	5+0
Fundamental of Machine Component Design 5th Ed.	9	11+0	9+0
ISO 9085:2002 and 6336 Standards	225	15+5	17+6
ANSI/AGMA 2010-D04 Standards	59	10+11	13+2

The design of an involute interference-free spur gear was carried out in a wider range (speed ratio of 1:1, 3:1, 5:1, 8:1 and 10:1 and power transmission of 1 kW, 10 kW, 100 kW, 500 kW and 1000 kW) using Microsoft Excel pages based on five approaches. Comparison was made by taking the ANSI/AGMA Standards as a reference. This is because the results of each of the gear design approach were verified using FEA and ANSI/AGMA standard provided closer result to the results of FEA. Therefore all results obtained from the approaches were normalized to ANSI/AGMA Standards. And a number of multiplication factors were obtained at a certain speed ratio for each of the approach considering failures criteria, see Tables 4.19 to 4.22. With the aid of the results of these tables, a number of formula to convert the modules obtained from the design approaches to ANSI/AGMA Standards was developed by using a curve fitting method. Therefore, a new multiplication factor can be obtained for any desired speed ratio by the following formulas developed in this study. In below equations “x” is the ratio of module obtained by the approaches to the module obtained by the ANSI/AGMA Standards. “y” is the new multiplication factor at any desired speed ratio.

Table 5.2. Formulas for obtaining multiplication factor based on bending fatigue failure

Design Approaches	Formulae of Multiplication Factor
Shigley J.E., 1985	$y = 0{,}0009.x^2 + 0{,}0007.x + 0{,}8602$
Budynas R.G. and Nisbett J.K., 2011	$y = 0{,}0013.x^2 - 0{,}0055.x + 0{,}6992$
Juvinall R.C. and Marshek K.M., 2011	$y = 0{,}0009.x^2 - 0{,}0078.x + 0{,}9918$
ISO 6336 Standards, 2006	$y = -0{,}0007.x^2 + 0{,}0266.x + 0{,}7003$

Table 5.3. Formulas for obtaining multiplication factor based on surface contact fatigue failure

Design Approaches	Formulae of Multiplication Factor
Shigley J.E., 1985	$y = -0{,}0009.x^2 + 0{,}147.x + 0{,}6767$
Budynas R.G. and Nisbett J.K., 2011	$y = -0{,}0005.x^2 + 0{,}0119.x + 0{,}7655$
Juvinall R.C. and Marshek K.M., 2011	$y = -0{,}0021.x^2 + 0{,}0329.x + 1{,}1104$
ISO 9085:2002 Standards, 2002	$y = -0{,}006.x^2 + 0{,}0844.x + 0{,}9699$

Based on the previous studies in Chapter 2, various works have been made using different kind of approaches without indicating the reason for selection of the approaches. In this case there will be confusion for researches that are not aware of success or loss gained using the approaches. Therefore the result of this work eliminates the confusion for researches.

This study proposes to use the easier and the most appropriate approach provided in the common text books considering the verified results of FEA, if there is no obligation to use ISO or ANSI/AGMA Standards. Because these standards are more challenging, time consuming and include complicated equations. Multiplication factors for the conversion of text books results to the verified results were developed. Now, the results obtained by text books can be converted to the standards with the aid of multiplication factors developed in this study. As a result of these, gear designers do not have to deal with the computational load of the standards.

Comparison of the approaches was carried out considering the gear fatigue failures of both bending and surface contact respectively. The following text proposes the easiest and effective design approaches for each of the gear failure criteria. And conclusions have been drawn based on these considerations.

Conclusions based on bending fatigue failure;

- Ø The smallest module considering the FEA verifications is obtained by the approach given by Budynas and Nisbett (2011). This approach is very simple and easy to understand but does not handle the design variables broadly. Therefore it is only recommended for where the reliability and life cycle of gear is not important. In this case this approach may be used for quick estimations.
- Ø ISO Standard is the second better approach but in order to eliminate the computational load of it and introduce a clear and simple gear design, Shigley’s (1985) approach is suggested which is the third better approach. This approach is also found to be in a better agreement with the reference one, ANSI/AGMA Standards.

- However if there is an obligation to use a standard, ISO 6336 Standard (2006) is recommended instead of ANSI/AGMA Standard for a better design. But when two standards are compared it is seen that ANSI/AGMA provides simpler and more understandable approach than ISO. In this case a designer can use ANSI/AGMA, and then can convert the result to ISO easily by using a multiplication factor provided in this study.
- Since the Juvinall and Marshek's (2011) approach gives the largest modules than the numerical values obtained by FEA, it is not recommended to use. However, if a spur gear is desired to be designed based on ANSI/AGMA Standards, Juvinall and Marshek's" approach may be used instead of ANSI/AGMA since it gives equal modules. This allows the designer to make an easy and quick design using empirical equations of "Juvinall and Marshek", and use the appropriate conversion factor to convert the results with a minor error to ANSI/AGMA standards.

Conclusions based on surface contact fatigue Failure;

- The results based on surface contact fatigue failure indicated that Shigley's (1985) approach for the gear design gives modules (m) that matches to the results obtained by FEA. This approach gives the smallest module. But although the input parameters are kept identical the effect of design factor of safety is not as much as other approaches. Therefore it may not be preferred. In this case the second better approach may be used that is given by Budynas and Nisbett's (2011).
- As mentioned above Budynas and Nisbett's (2011) approach was suggested to use for simple quick estimation designs which is more suitable for novice designers and learners. After the design is carried out, the designers can use a multiplication factor to convert the results. But expert designers should use the third better approach which is ANSI/AGMA Standards.

- However if there is an obligation about using a standard, ANSI/AGMA 2101-D04 Standard (2004) should be used instead of ISO 9085:2002 Standard (2002) as it gave matching results to the results obtained from FEA. In addition to this, ISO Standards are more complicated and time consuming than ANSI/AGMA. This study has indicated that considering its disadvantages there is no need to use ISO Standards for a spur gear design based on surface contact fatigue failure.

As the module is the one of the most important design parameter face width is the second more important design output. The face width have also to be taken into account. This is because both module and face width determine the overall size of a gear which directly determines the cost. The effect of face width was presented in Section 4.5. And the trend was found to be the same as the results of module.

The above findings also indicated the best gear design approach that gives the matching results obtained from FEA. Interestingly, the best gear design approaches gave the smallest design values of module (m) and face width (F). The findings may also lead and guide designers, when selecting the appropriate gear design approach if they are aiming to optimize the dimensions of the spur gear.

Briefly, this study may serve as a guideline for a designer who deals with the design of an involute spur gear. If a designer concerns with light weighted applications, the overall size of a gear is important as well as material usage that are objectives of optimization. On the other hand spur gear design is the subject of almost all machine design courses. And it is important to introduce clear, easy to understand and reliable design approach for learners and students. Consequently, the results of this work interests both expert and novice designers and learners.

REFERENCES

SGIHLEY J.E., 1985. Mechanical Engineering Design: First Metric Edition. Mcgraw-Hill, 698 pages.

BUDYNAS R.G. and NISBETT J.K., 2011. Shigley's Mechanical Engineering Design. Ninth Edition, McGraw-Hill, 1120 pages.

JUVINALL R.C., MARSHEK K.M., 2011. Fundamentals of Machine Component Design. Wiley 5th Edition, 928 pages.

MOTT R.L., 2003. Machine Elements in Mechanical Design. 4th Edition, Prentice Hall, 944 pages.

RADZEVICH S.P., 2012. Dudley's Handbook of Practical Gear Design and Manufacture. Second Edition, CRC Press, Taylor & Francis Group, 878 pages.

CHILDS P.R.N., 2013. Mechanical Design Engineering Handbook. First Edition, Butterworth-Heinemann Ltd, 856 pages.

UGURAL A.C., 2003. Mechanical Design an Integrated Approach 1st Edition. McGraw Hill, 864 pages.

JELASKA D.T, 2012, Gears and Gear Drives. 1st Edition, Wiley, 462 pages.

BABALIK F.C., 2010, Makine Elemanları ve Konstrüksiyon Örnekleri. Dora Basım Yayınevi, 4. Baskı, 860 sayfa.

GÖKÇEK M., 2012, Mechanical Engineering. First Published by InTech, 670 pages.

KAPELEVICH A.L., 2013. Direct Gear Design. CRC Press, Taylor & Francis Group. 324 pages.

ASHBY M.F., 2010. Material Selection in Mechanical Design. Butterworth-Heinemann 4th Edition, 640 pages.

ISO Standards 6336 – Part 1, 2006. Calculation of load capacity of spur and helical gears – Basic principles, introduction and general influence factors. 109 pages, Switzerland.

ISO Standards 6336 – Part 3, 2006. Calculation of load capacity of spur and helical gears – Calculation of tooth bending strength. 42 pages, London, UK.

ISO Standards 6336 – Part 5, 2003. Calculation of load capacity of spur and helical gears – Strength and quality of materials. 43 pages, Switzerland.

ISO Standards 6336 – Part 6, 2004. Calculation of load capacity of spur and helical gears – Calculation of service life under variable load. 23 pages, Switzerland.

ISO 9085:2002, 2002. Calculation of load capacity of spur and helical gears – Application for industrial gears. 57 pages, Switzerland.

ANSI/AGMA 2101-D04 Standards, 2004. Fundamental rating factors and calculation methods for involute spur and helical gear teeth. 59 pages, Virginia, USA.

AGMA 908-B89, 1989. Geometry factors for determining the pitting resistance and bending strength of spur, helical and herringbone gear teeth. 73 pages, Virginia, USA.

KAWALEC A., WIKTOR J., and CEGLAREK D., 2006. Comparative Analysis of Tooth-Root Strength Using ISO and AGMA Standards in Spur and Helical Gears With FEM-based Verification. Journal of Mechanical Design, Vol. 128, 5:1141-1158.

TIWARI S.K. and JOSHI U.K., 2012, Stress Analysis of Mating Involute Spur Gear teeth. International Journal of Engineering Research and Technology, Volume 1, Issue 9.

KARAVEER V., MOGREKAR A. and JOSEPH T.P.R., 2013. Modeling and Finite Element Analysis of Spur Gear. International Journal of Current Engineering and Technology, Vol.3 No.5:2104-2107.

SHINDE S.P., NIKAM A.A. and MULLA T.S., 2009. Static Analysis of Spur Gear Using Finite Element Analysis. IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE) ISSN: 2278-1684, PP: 26-31.

AMBADE V.V., VANALKAR A.V. and GAJBHIYE P.R., 2013. Involute Gear Tooth Contact and Bending Stress Analysis, International Journal of Computational Engineering Research, Vol.03, Issue, 8: 30-36.

FETVACI M.C. and İMRAK C.E., 2004. Düz Dişli Çarkların Sonlu Elemanlar Metodu ile Modellenmesi. Sigma Journal of engineering and natural sciences, Vol. 19, 2, 199-203.

GUPTA B., CHOUBEY A. and VARDE G.V., 2012. Contact Stress Analysis of Spur Gear. International Journal of Engineering Research & Technology (IJERT) Vol. 1 Issue 4, 7 pages.

LI S., 2007. Finite Element Analyses For Contact Strength and Bending Strength of A Pair of Spur Gears With Machining Errors, Assembly Errors and Tooth Modifications. Mechanism and Machine Theory 42, 88–114.

GURUMANI R. and SHANMUGAM S., 2011. Modeling and Contact Analysis of Crowned Spur Gear Teeth. Engineering MECHANICS, Vol. 18, No. 1, p. 65–78.

PATEL I. and MURTHY M.S., 2013. Comparison of Bending Stresses for Different Number of Teeth of Spur Gear Obtained Using MATLAB Simulink with AGMA and ANSYS. International Journal of Engineering Trends and Technology (IJETT) - Volume4, Issue7, p. 3141-3144.

HWANG S.C., LEE J.H., LEE D.H., HAN S.H. and LEE K.H., 2013. Contact Stress Analysis for a Pair of Mating Gears. Mathematical and Computer Modelling 57, p. 40–49.

SANKAR S. and NATARAJ M., 2011. Profile Modification - A Design Approach for Increasing the Tooth Strength in Spur Gear. The International Journal of Advanced Manufacturing Technology, 55:1-10.

PARTHIBAN A., RAJU P.R., SREENIVASULU V., RAO P.D. and KIRAN C.U., 2013. Profile Modification for Increasing the Tooth Strength in Spur Gear using CAD & CAE. International Journal of Innovations in Engineering and Technology, Vol.2, Issue 1, p. 231-241.

PEDERSEN N.L., RAJU P.R., SREENIVASULU V., RAO P.D. and KIRAN C.U., 2010. Improving Bending Stress in Spur Gears Using Asymmetric Gears and Shape Optimization. Mechanism and Machine Theory 45, p.1707–1720.

KAWALEC A. and WIKTOR J., 2008. Tooth Root Strength of Spur and Helical Gears Manufactured with Gear-Shaper Cutters. Journal of Mechanical Design, Vol.130, 5 pages.

MARKOVIĆ K. and FRANULOVIĆ M., 2011. Contact Stresses in Gear Teeth Due to Tip Relief Profile Modification, Engineering Review, Vol. 31-1, p. 19-26.

DHAVALE A.S. and UTPAT A., 2013. Study of Stress Relief Features at Root of Teeth of Spur Gear. International Journal of Engineering Research and Applications Vol. 3, Issue 3, p.895-899.

JEBUR A.K., KHAN I.A. and NATH Y., 2011. Numerical and Experimental Dynamic Contact of Rotating Spur Gear. Modern Applied Science, Vol. 5, No. 2, p. 254-263.

MENDI F., BAŞKAL T., BORAN K. and BORAN F.E., 2010. Optimization of Module, Shaft Diameter and Rolling Bearing for Spur Gear through Genetic Algorithm. Expert Systems with Applications, 37, 8058–8064.

MARJANOVIC N., ISAILOVIC B., MARJANOVIC V., MILOJEVIC Z., BLAGOJEVIC M. and BOJIC M., 2012. A Practical Approach to the Optimization of Gear Trains with Spur Gears, Mechanism and Machine Theory, 53, p. 1–16.

GOLABI S., FESHARAKI J.J. and YAZDIPOOR M., 2014. Gear Train Optimization Based on Minimum Volume/Weight Design, Mechanism and Machine Theory, 73, p. 197–217.

GEREN N. and BAYSAL M.M., 2000. Expert System Development for Spur Gear Design. 9. International Conference on Machine Design and Production.

KAMBLE A.G., VENKATA R. R., POTDAR A.S., and LOKHANDE A.D., 2010. Prediction and Optimization of Spur Gear Pair by Response Surface Method. Journal of Advances in Engineering Science Section B (1), p. 23-28.

LI C., CHIOU H., HUNG C., CHANG Y. and YEN C., 2002. Integration of Finite Element Analysis and Optimum Design on Gear Systems. Finite Elements in Analysis and Design 38, p. 179-192.

GOLOGLU C. and ZEYVELI M., 2009. A Genetic Approach to Automate Preliminary Design of Gear Drives. Computers & Industrial Engineering, 57, p. 1043–1051.

SANCHEZ M.B., PEDRERO J.I. and PLEGUEZUELOS M., 2013. Critical Stress and Load Conditions for Bending Calculations of Involute Spur and Helical Gears. International Journal of Fatigue, 48, p. 28–38.

HUANG K. J. and SU H.W., 2010. Approaches to Parametric Element Constructions and Dynamic Analyses of Spur/Helical Gears Including Modifications and Undercutting. Finite Elements in Analysis and Design 46, p. 1106–1113.

URL : http://machinedesign.com/

CHALA, G., 1999. ISO 6336 VS AGMA 2001 GEAR RATING COMPARISON for INDUSTRIAL GEAR APPLICATIONS, Rexnord Industries, LLC.

BERG MANUFACTURING, Gear Reference Guide.

BECKMAN K.O., PATEL V.P., 2000. Review of API versus AGMA Gear Standards – Ratings, Data Sheet Completion, and Gear Selection Guidelines.

BIOGRAPHY

Çağrı UZAY was born in 14.08.1989 in Kahramanmaraş. He had lived in İstanbul, Diyarbakır, Kahramanmaraş, Kırşehir. He has lived in Adana since 2001. He graduated from Seyhan Rotary Anatolian High School in 2007 and enrolled at Mechanical Engineering Department of Çukurova University in the same year. He graduated with his Bachelor of Science degree in Mechanical Engineering Department with the 1st degree in 2012. He also completed the undergraduate program of Automotive Engineering with double major in the same year. He started to work in TEMSA GLOBAL A.Ş. as an R&D Engineer on June, 2012. In the same year on September he began his Master of Science education at Mechanical Engineering Department of Çukurova University. He had worked for about two years in TEMSA GLOBAL A.Ş and leaved on January, 2014 and appointed as a Research Assistant to Kahramanmaraş Sütçü İmam University, Faculty of Engineering and Architecture, Department of Mechanical Engineering, Division of Construction and Manufacturing. Since 2014, he has been working as a Research Assistant at Mechanical Engineering Department of Çukurova University for carrying out his Master of Science education.

APPENDIX

Tablo 22.12 $K_{H\alpha}$ ve $K_{F\alpha}$ alın yük dağılım faktörleri

<table>
<tr><td colspan="3">K_A . F_t/b</td><td colspan="7">> 100 N/mm</td><td>≤ 100 N/mm</td></tr>
<tr><td colspan="3">Dişli Kalitesi →</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td><td>< 6</td></tr>
<tr><td rowspan="2">Sertleştirilmiş</td><td>Düz</td><td>$K_{H\alpha}$
$K_{F\alpha}$</td><td colspan="2">1,0</td><td>1,1</td><td>1,2</td><td colspan="4">$1/Z_\varepsilon^2 \geq 1,2$
$1/Y_\varepsilon \geq 1,2$</td></tr>
<tr><td>Helisel</td><td>$K_{H\alpha}$
$K_{F\alpha}$</td><td>1,0</td><td>1,1</td><td>1,2</td><td>1,4</td><td colspan="4">$\varepsilon_\alpha/\cos^2\beta_b \geq 4$</td></tr>
<tr><td rowspan="2">Sertleştirilmemiş</td><td>Düz</td><td>$K_{H\alpha}$
$K_{F\alpha}$</td><td colspan="3">1,0</td><td>1,1</td><td>1,2</td><td colspan="3">$1/Z_\varepsilon^2 \geq 1,2$
$1/Y_\varepsilon \geq 1,2$</td></tr>
<tr><td>Helisel</td><td>$K_{H\alpha}$
$K_{F\alpha}$</td><td colspan="2">1,0</td><td>1,1</td><td>1,2</td><td>1,4</td><td colspan="3">$\varepsilon_\alpha/\cos^2\beta_b \geq 4$</td></tr>
</table>

Figure A.1. Transverse load factors for tooth bending stress and surface stress (Babalık F.C., 2010)

	Profil Kaydırma Faktörü x																				
z ; z_n	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	+0.1	+0.2	+0.3	+0.4	+0.5	+0.6	+0.7	+0.8	+0.9	+1.0	+1.1	+1.2	+1.3	+1.4
7												2.84									
8											2.98	2.69	2.47								
9											2.84	2.60	2.40	2.22							
10										2.99	2.73	2.52	2.34	2.18							
11									3.15	2.87	2.65	2.46	2.30	2.16	2.05						
12									2.03	2.79	2.58	2.41	2.27	2.14	2.04						
13									2.93	2.72	2.53	2.38	2.24	2.12	2.03	1.96					
14							3.36	3.10	2.86	2.66	2.48	2.34	2.22	2.11	2.03	1.95					
15							3.25	3.01	2.79	2.60	2.44	2.31	2.20	2.10	2.02	1.95	1.89				
16						3.45	3.16	2.95	2.74	2.56	2.42	2.29	2.18	2.09	2.02	1.95	1.89				
17						3.35	3.09	2.88	2.69	2.53	2.39	2.27	2.17	2.08	2.01	1.95	1.89	1.85			
18					3.53	3.26	3.02	2.82	2.65	2.50	2.37	2.26	2.16	2.08	2.01	1.95	1.90	1.86			
19				3.72	3.44	3.20	2.96	2.78	2.61	2.47	2.35	2.24	2.15	2.07	2.01	1.95	1.90	1.87	1.83		
20				3.62	3.35	3.12	2.91	2.74	2.58	2.45	2.33	2.23	2.14	2.07	2.01	1.96	1.90	1.87	1.84		
21				3.53	3.28	3.07	2.87	2.70	2.55	2.43	2.32	2.22	2.14	2.06	2.01	1.95	1.91	1.87	1.84	1.82	
22				3.45	3.20	3.01	2.83	2.67	2.52	2.41	2.80	2.21	2.13	2.06	2.00	1.95	1.91	1.88	1.85	1.83	
23			3.64	3.38	3.15	2.96	2.80	2.64	2.50	2.39	2.29	2.20	2.12	2.06	2.00	1.95	1.91	1.88	1.85	1.83	1.82
24			3.55	3.30	3.10	2.92	2.75	2.61	2.48	2.37	2.28	2.19	2.12	2.06	2.00	1.95	1.91	1.88	1.86	1.84	1.83
25		3.73	3.45	3.25	3.05	2.88	2.72	2.58	2.46	2.36	2.27	2.19	2.12	2.05	2.00	1.95	1.92	1.88	1.86	1.84	1.83
30	3.61	3.35	3.18	3.01	2.85	2.72	2.60	2.48	2.38	2.30	2.22	2.16	2.10	2.04	2.00	1.96	1.93	1.90	1.88	1.85	1.85
40	3.15	3.00	2.86	2.75	2.63	2.54	2.45	2.37	2.30	2.24	2.18	2.13	2.08	2.04	2.01	1.97	1.95	1.93	1.91	1.90	1.89
50	2.90	2.78	2.68	2.59	2.50	2.43	2.36	2.31	2.25	2.20	2.15	2.11	2.07	2.03	2.02	1.98	1.97	1.94	1.93	1.92	1.91
60	2.75	2.65	2.57	2.50	2.42	2.37	2.32	2.25	2.22	2.17	2.13	2.10	2.08	2.04	2.02	1.99	1.98	1.96	1.94	1.94	1.93
100	2.46	2.40	2.35	2.32	2.26	2.24	2.21	2.17	2.15	2.12	2.10	2.08	2.06	2.04	2.03	2.01	2.00	1.99	1.98	1.98	1.97
200	2.27	2.24	2.21	2.19	2.17	2.15	2.14	2.12	2.10	2.10	2.06	2.07	2.05	2.04	2.04	2.02	2.02	2.01	1.98	2.00	2.00
400	2.17	2.15	2.14	2.13	2.12	2.11	2.10	2.09	2.08	2.08	2.08	2.07	2.06	2.06	2.05	2.04	2.04	2.04	2.03	2.03	2.03
∞	2.07	2.07	2.07	2.07	2.07	2.07	2.07	2.07	2.07	2.07	2.07	2.07	2.07	2.07	2.07	2.07	2.07	2.07	2.07	2.07	2.07

Figure B.1. Tooth form factor (Babalık F.C., 2010)

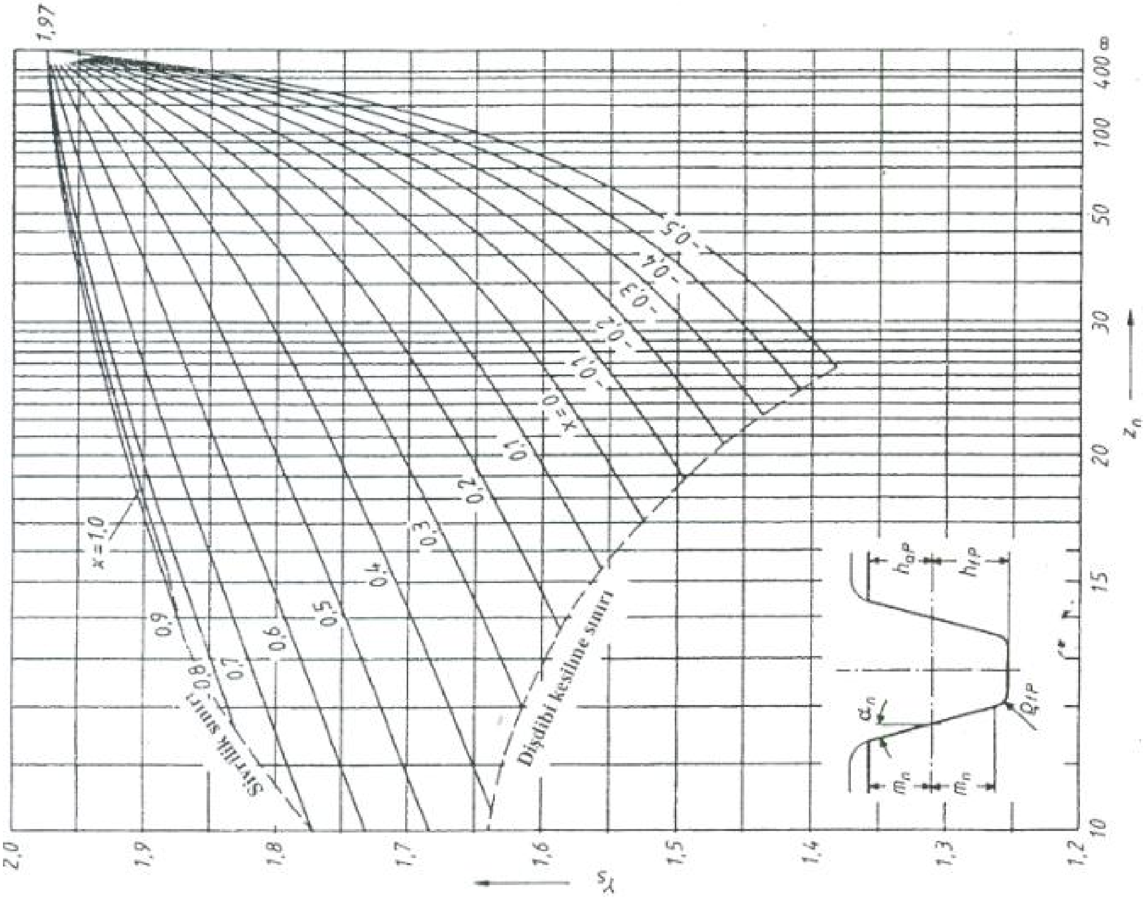


Figure C.1. Stress correction factor (Babalık F.C., 2010)

Material	Type	Abbreviation
Normalized low carbon steels / cast steels	Wrought normalized low carbon steels	St
	Cast steels	St (cast)
Cast iron materials	Black malleable cast iron (perlitic structure)	GTS (perl.)
	Nodular cast iron (perlitic, bainitic, ferritic structure)	GGG (perl., bai., ferr.)
	Grey cast iron	GG
Through-hardened wrought steels	Carbon steels, alloy steels	V
Through-hardened cast steels	Carbon steels, alloy steels	V(cast)
Case-hardened wrought steels		Eh
Flame or induction hardened wrought or cast steels		IF
Nitrided wrought steels / nitriding steels / through-hardening steels, nitrided	Nitriding steels	NT(nitr.)
	Through hardening steels	NV (nitr.)
Wrought steels, nitrocarburized	Through hardening steels	NV (nitrocar.)

Figure D.1. Materials (ISO 6336-Part 1 Standards, 2006)